N. A. Rynin

INTERPLANETARY FLIGHT AND COMMUNICATION

Volume II, No. 5

Theory of Rocket Propulsion

TRANSLATED FROM RUSSIAN

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1. INTRODUCTION

Man has long dreamt of overcoming terrestrial gravitational forces and of exploring the Sun, Moon and other planets; but only nowadays do these ideas about interstellar flights have better prospects of being realized, due to the progress achieved in both science and technology.

Let us briefly examine a few methods by means of which some inventors have suggested that man could leave the Earth and start out for interplanetary space:

1. A shot from a cannon (projects of Jules Vernes, Graffigny, Valier and others). The shell containing the cabin was to be placed in the gun barrel or inside a volcanic crater and ejected upward with a shot. However, it would be impossible to put a human being into such a shell because his body could not tolerate the overload built up by excessive acceleration. Besides, to overcome the gravitational forces and the air drag one would need a cannon of really fantastic dimensions.

2. Ejection by a centrifugal device (Graffigny's project). The missile containing the cabin was to be attached to the circumference of a large wheel and the wheel rotated to obtain the necessary velocity. At a given moment the missile was to be separated from the wheel and hurled into outer space. This idea, though theoretically correct, cannot yet be realized (using the existing materials) because it is impossible to construct a wheel of the required dimensions that will withstand such considerable velocities.

3. The reactive craft. Flight of such a craft is based upon the principle of rocket flight. Inside the craft the fuel is burnt and the gaseous products of combustion discharged rearward through the nozzle. This provides thrust, by means of reaction, and consequently the craft moves in a direction opposite to that of the ejected gases.

Most of the scientists and engineers who have worked on spacecraft
2 projects have based their propulsion designs on this principle of action and reaction. One should mention names such as Tsiolkovskii, Esnault-Pelterie, Goddard, Oberth, Valier, Hohmann, Tsander, Hanswindt and many others who have contributed to the field of rocket motion. From now on I will restrict myself entirely to this theory because at the present stage of development of science and technology it is the only one that can be used for extraterrestrial space flights.

Moreover, I will present only the main theoretical assumptions concerning thrust conditions of rocket flight. Our chief consideration will be the problem of launching a rocket and its descent. Flight conditions, especially orientation in space and navigation, are of secondary importance in this work and therefore are just mentioned.

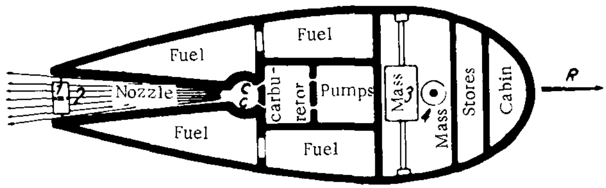

FIGURE 1. Reactive spacecraft

2. THE REACTIVE CRAFT

A general diagram of a reactive craft is shown in Figure 1. The fuel is pumped into the mixing chamber (the carburetor) and then into the combustion chamber (c. c.), where it is burned explosively. The gaseous products of combustion are discharged rearward through the nozzle, a gradually widening tube. The passengers' compartment and the provision stores are situated at the front of the craft. The necessary thrust is created by reaction (R) to the hot gaseous jet. For flight navigation and control, vanes inserted into the jet may be used as an elevator (1) and a rudder (2). Masses (3 and 4), adjustable in their positions along two mutually perpendicular axes,may serve the same purpose.

Many reactive spaceship designs were suggested by Tsiolkovskii, Goddard, Hanswindt, Gussali, Hohmann, Oberth, Valier, Tsander and others. I will describe some of the most advanced designs in general.

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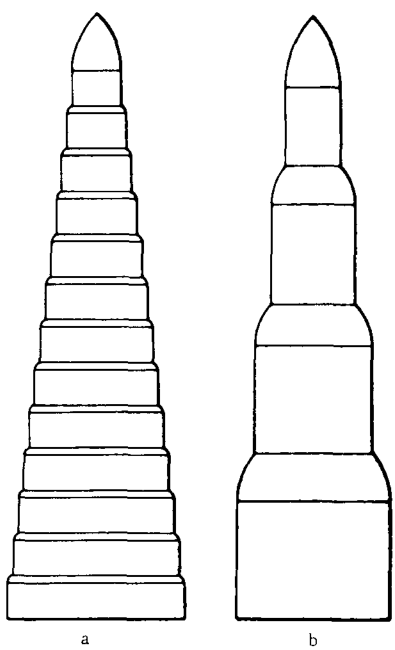


FIGURE 2. Goddard's multistage rockets

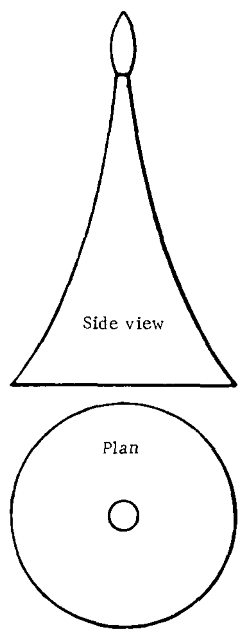


FIGURE 3. Hohmann's rocket

G o d d a r d ' s m u l t i s t a g e r o c k e t (Figure 2a and 2b). The rocket consists of a number of fuel-containing propellant tanks which are successively jettisoned as they become empty, gradually increasing the rocket velocity. Figure 2a shows a rocket built of many small parts, while Figure 2b presents a rocket consisting of a few large stages. The cabin is situated at the rocket head.

3 H o h m a n n ' s r o c k e t. The cabin is placed at the rocket head (Figure 3, top). Most of the rocket volume is occupied by the fuel, which reduces the rocket mass gradually, as it burns.

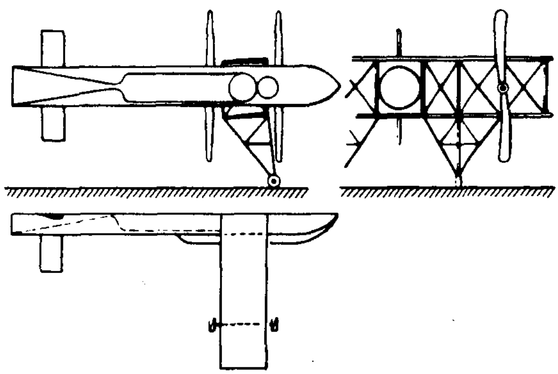


FIGURE 4. Tsander's winged rocket

T s a n d e r ' s w i n g e d r o c k e t (Figure 4). The fuselage of the craft is a conventional rocket. In addition, its configuration consists of typical airplane components to facilitate the launching, such as wings, stabilizers, propellers and landing gear. It may have skis for landing on land or water. Wings enable the craft to glide in the atmosphere on its way back. It must be pointed out that Tsander's design repeats, in general, the idea of the French engineer Laurent.

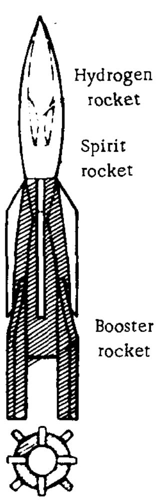


FIGURE 5. Oberth's compound rocket

O b e r t h ' s c o m p o u n d r o c k e t (Figure 3). The rocket consists of three parts. The small one contains hydrogen fuel and the capsule with a parachute. Alcohol serves as the second-stage propellant, whereas the third stage is an additional booster rocket. The
4 launching process occurs under the following sequence. The booster motor starts functioning and is jettisoned when its fuel is consumed. Then the second stage burns and when it is its turn to be jettisoned, the small stage takes over. Finally, while descending, the third stage is separated too, and the capsule alone performs a soft parachute landing.

T s i o l k o v s k i i ' s r o c k e t. Konstantin Tsiolkovskii is a pioneer in the field of reactive interplanetary craft. It is impossible to describe here in detail each of his numerous research works in this field. I will only mention that he suggested eight different types of reactive craft: the first is a straight-nozzle rocket-engine design (1903); the second is a curved-nozzle engine (1914); the third is an improvement of the first variant (1915); the fourth is an experimental rocket (1917); the fifth type is a multi-stage manned spacecraft (1917); the sixth is a portable rocket (a bag configuration); the seventh is a moon-flight rocket and the eighth is an improved third type (1927).

This last variant seems to be the most advanced and is presented in Figure 6. The spacecraft is designed for minimum atmospheric resistance. The compartments inside are situated as follows (from right to left): provision stores, pilot compartment, flight-control mass, fuel tanks (liquid hydrogen and oxygen), fuel-mixing chamber, pumps, batteries, igniter, nozzle and control vanes. Flight observations
5 are made by means of a periscope. Another periscope actuates (by light reflection) the electrical mechanism which automatically adjusts the position of a heavy mass (flight-course controlling device).

(4)

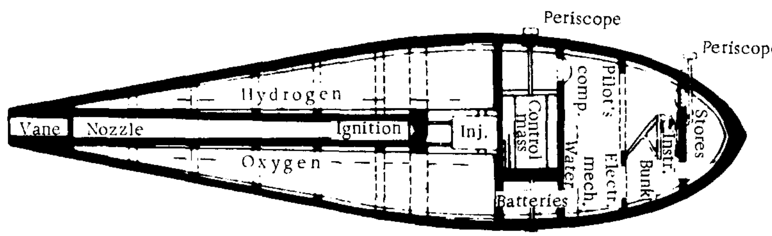


FIGURE 6. Tsiolkovskii's rocket

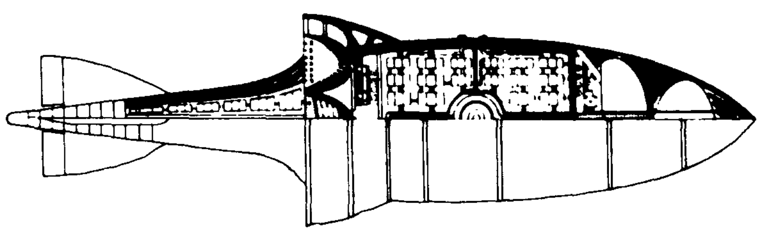


FIGURE 7. Oberth-Valier rocket

Oberth-Valier rocket (Figure 7). Instead of one central nozzle it has a series of small nozzles situated on the circumference of the spacecraft's central part. Stabilizers and control fins are fixed at its tail. The interior of the craft contains fuel tanks, compartments, instrumentation and provision stores.

3. THE EQUATIONS OF ROCKET MOTION

Basic theoretical principles of reactive spacecraft flight were published by Tsiolkovskii in 1903. Contributors to this theory of reactive motion were also made by Esnault-Pelterie, Goddard, Oberth, Hohmann, Lorenz, Lademann,* Scherschevsky and Valier. One should distinguish between the following principal flight phases when studying this theory:

a) The general case, when jet reaction forces, gravitational attraction and atmospheric resistance act upon the rocket; this occurs during the takeoff and the descent of the craft.

b) Flight outside the atmosphere of the planet but within the limits of its gravitational field; this is the case when the rocket has actually left the atmospheric medium, but remains under the influence of gravity.

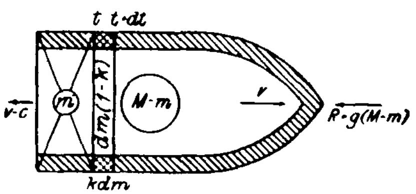


FIGURE 8. Deriving the equation of rocket motion

c) Motion imparted to the craft by the jet reaction while the forces of both atmospheric resistance and gravity are absent; this is a theoretical case approaching in its concept the case when the rocket passes the point of equilibrium between the attractions of Earth and the Moon.

d) Free flight, when the rocket is not subject to any reaction force of gases and follows Kepler's trajectory due only to gravitational forces.

I will first present the rocket-motion equation for the case (a), i. e., when jet reaction, aerodynamic drag and gravity act upon the craft in the course of its takeoff.

I will introduce the following notations (Figure 8):

M — the initial mass of the rocket;

m — the mass ejected up to the moment t. This mass consists of the gaseous products of combustion and the shell fragments of the propellant tanks which are detached just as the propellant is consumed;

v — rocket velocity at moment t;

6 c — velocity of the ejected mass;

R — air-drag force;

g — gravitational acceleration;

dm — mass ejected in interval dt;

k — constant ratio of the shell mass (propellant tanks) to the mass of propellant. The tanks are jettisoned with zero velocity relative to the remaining rocket mass;

dv — rocket velocity increase during interval dt.

I will derive the rocket-motion differential equations according to Newton's third law, first using the condition that the momentum at the moment t equals the momentum at the moment t + dt plus the impulse of the atmospheric resistance and gravitational forces:

$$(M-m)\,v = dm\,(1-k)\,(v-c) + vkdm + (M-m-dm)\,(v+dv) + [R + g\,(M-m)]\,dt.$$

* [The Russian original reads Lindemann which is obviously a misprint.]

Neglecting the second-order terms, we obtain:

$$c\,(1-k)\,dm = (M-m)\,dv + [R + g\,(M-m)]\,dt\,. \tag{1}$$

In solving this equation the following conditions should be fulfilled:

1) the initial mass M, which consists mainly of propellant, should not be too large and consequently,

2) the propellant mass m, ejected up to the moment t, should be minimal;

3) optimum flight velocity v should be chosen so that the resultant combined air drag and gravitational forces have the least effect. At a very high velocity v considerable atmospheric resistance is coupled with rapidly diminishing gravitational effect (i. e., its impulse is small); on the other hand, at low v air drag is small but the deleterious effect of gravity will increase;

4) the highest possible efficiency of fuel combustion, i. e., the ejection velocity (c) of its gaseous products should be the highest obtainable;

5) in the case of a manned flight the rocket acceleration should not exceed the limit of human endurance.

Before solving the general rocket-motion equation it is necessary to clarify several specific questions on which the terms of this equation depend, in particular:

a) gravitational forces (as if an envelope of gravitational attraction had to be pierced);

b) atmospheric resistance (as if an envelope of aerodynamic drag force had to be pierced);

c) propellant energy;

d) physiological effects of acceleration.

I will now treat each factor separately.

7 4. THE GRAVITATIONAL ENVELOPE

A rocket undertaking an interplanetary flight must develop a certain minimum velocity to overcome the Earth's attraction and air-drag forces (i. e., as if breaking through the gravitational and atmospheric envelopes of the Earth). This section will show how to derive the expression for the gravitational forces of the Earth and other planets of the solar system, how to find the work and the velocity needed for a rocket to free itself from the gravitational links and take off into space.

a) Work by gravity

I will derive the expression for the work necessary to move the rocket out of the terrestrial gravitational field.

The notation r will stand for the Earth's radius and g_0 for the gravitational acceleration on its surface.

Let a rocket of mass m be launched up to a certain altitude so that its distance from the Earth's center is r_1. The corresponding gravitational acceleration will be

$$g_1 = g_0 \cdot \frac{r^2}{r_1^2};$$

the work done by the rocket will be:

$$T = \int_r^{r_1} m\, g_0 \frac{r^2}{r_1{}^2}\, dr = m g_0 r^2 \left(\frac{1}{r} - \frac{1}{r_1}\right). \qquad (2)$$

If $mg_0 = 1$ kg, i. e., the rocket weight is 1 kg and $r = 6.371 \cdot 10^6$ m, then the work needed to move a body to infinity, i. e., out of the terrestrial gravitational field, will be:

$$T_\infty = 1 \cdot (6.37 \cdot 10^6)^2 \left(\frac{1}{6.37 \cdot 10^6} - \frac{1}{\infty}\right) = 6.37 \cdot 10^6\ \text{kg} \cdot \text{m}.$$

I will represent graphically the variation in the rocket weight, i. e., the force of its attraction to the Earth with the increase of distance between the two. In Figure 9 a circle of radius r describes the Earth. The axis of abscissas is OU. The axis of ordinates, perpendicular to OU, is drawn through the end of the radius and segment AB on it represents (on an arbitrary scale) the rocket weight on the surface of the Earth. I will use the ratio $\frac{a}{r^2}$ for it, where a is a number dependent on the Earth and rocket masses.

Then in a distance of two Earth radii the rocket weight will be $\frac{a}{(2r)^2}$,
in a distance of three r, $\frac{a}{(3r)^2}$ and so on. In a distance x the weight of
8 the rocket will be $\frac{a}{x^2}$. A curve BCDEFV . . . traced through these ordinate values approaches asymptotically the OU-axis. I will prove that the area limited by the OU-axis, ordinate AB and curve BV (extended to infinity) equals the area of the rectangle OABM.

I will examine an element of this area which is located at distance x from the center O and of width dx.

Its area equals:

$$dp = \frac{a}{x^2} \cdot dx.$$

By integration from r to ∞, the entire sought area is obtained:

$$\int_r^\infty \frac{a\,dx}{x^2} = a\left(-\frac{1}{x} + C\right)_r^\infty = \frac{a}{r} = r \cdot \frac{a}{r^2} = OA \cdot AB,\ \text{which was to be proved.}$$

This area (OABM) represents the work of the gravitational force which must be overcome in order to move a body outside the terrestrial gravitational field. It equals the Earth's radius multiplied by the body weight (as follows from the formula $r \cdot \frac{a}{r^2}$).

A rocket traveling from the Earth to the Moon will pass at a certain distance (x) from the Earth a neutral zone (where the forces of the attraction of the Moon and Earth are equal) and will then fall onto the Moon surface. I will denote the Earth mass by M, the Moon mass by $M_1 = 0.01228\,M$, the rocket mass by m and the approximate distance between the Moon and the Earth as 384,300 km. Thus Newton's law and the condition of equal attraction forces give

$$k\,\frac{Mm}{x^2} = k\,\frac{M_1 m}{(384{,}300-x)^2}\,;$$

here k is the universal gravitational constant. From this we obtain

$$x = 345{,}963\ \text{km}.$$

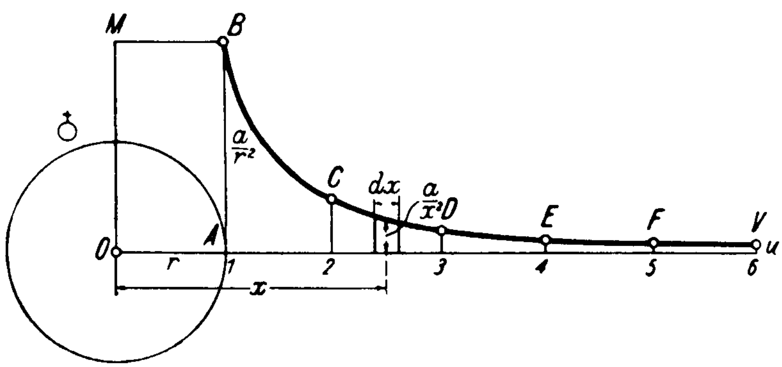


FIGURE 9. Variation in rocket weight with increase of its distance from the Earth

9 b) Flight velocities

I will now find the velocity which a rocket needs in order to escape the gravitational field of a planet through a horizontal launch.

A planet section is shown on Figure 10. A rocket with a velocity $v = AB$ m/sec is launched horizontally from point A of its surface. The planet radius is $AC = r$.

If the gravitational effect were absent, the rocket would reach point B in one second. But because of the gravitational attraction the rocket will approach the center C by a distance $BD = g_0/2$, which is half the acceleration of gravity. In order to prevent the missile from falling back to the planet surface and to make it circle the planet (along a path of which arc AD is a segment) the following condition is to be fulfilled:

$$AB^2 = BC^2 - AC^2$$

or

$$v^2 = \left(r + \frac{g_0}{2}\right)^2 - r^2 = g_0 r + \frac{g_0^2}{4}.$$

If we neglect $\frac{g_0^2}{4}$, we obtain:

$$v = \sqrt{g_0 \cdot r}. \qquad (3)$$

As long as the rocket velocity is the one derived from condition (3), the rocket will circle the planet as a satellite. An increase in v will convert the orbit into an ellipse, which will become more prolate at higher velocities.

Finally, at a certain velocity the rocket will be launched into space on a no-return parabolic trajectory. This velocity is determined by equating the work of the gravitational force to the change in the kinetic energy of the rocket:

$$\frac{m(v^2 - v_1^2)}{2} = mg_0 r^2 \left(\frac{1}{r} - \frac{1}{r_1}\right).$$

For moving the rocket to infinity ($r_1 = \infty$) and stopping it completely these ($v_1 = 0$), we obtain the initial takeoff velocity:

$$v_0 = \sqrt{2g_0 r} = v\sqrt{2}. \qquad (4)$$

Table 1 presents the values of v and v_0 for different planets of the solar system.

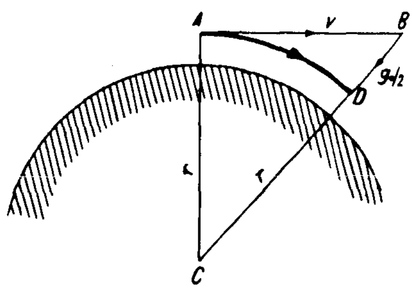


FIGURE 10. The flight velocity of a rocket

10 For example, a rocket launched from the Earth with a velocity of 7,906 m/sec will circle it; with velocities between 7,900 and 11,180 m/sec it will trace an ellipse and at $v \geqslant 11{,}180$ m/sec the rocket will escape on a no-return parabolic trajectory.

TABLE 1. Rocket-launching velocities from different planets

Planet	Radius of planet, km	Acceleration of gravity g_0, m/sec^2	$g_0 r$, m^2/sec^2	Circular velocity $v = \sqrt{g_0 r}$, m/sec	Parabolic velocity $v_0 = v\sqrt{2}$, m/sec	Remarks
Sun	695,445	269	187,074,705,000	432,521	611,628	* For planetoids $g_0 = 9.81\frac{r_n}{r_E}$, for example, for Pallas $g_0 = 9.81\frac{243}{6371} = 0.37$, assuming they have equal densities
Mercury	2,420	5.1	12,342,000	3,573	5,052	
Venus	6,087	8.42	51,252,000	7,160	10,126	
Earth	6,371	9.81	62,500,000	7,906	11,180	
Moon	1,736	1.62	2,812,000	1,676	2,370	
Mars	3,391	3.69	9,122,000	3,620	5,119	
Atalanta	15	0.023*	345	19	27	
Pallas	243	0.37*	90,000	300	424	
Ceres	402	0.62*	249,240	500	707	
Jupiter	71,368	24.94	1,779,818,000	42,188	59,662	
Saturn	61,513	10.64	654,498,000	25,583	36,179	
Uranus	24,292	8.60	208,911,000	14,453	20,439	
Neptune	28,017	9.60	268,963,000	16,402	23,196	

The trajectory of a rocket traveling from the Earth to another planet will be divided into three distinct phases: 1) the powered motion, during which the rocket engine functions and up to the moment when the necessary (almost parabolic) velocity is obtained; 2) the free flight (coasting) along an elliptical path (a Kepler curve) to the gravity field limits of the terminal planet and 3) descent on this planet decelerating by means of the *retro-thrust* of the *rocket engine* (both glide and parachute landings can also be realized).

A rocket launched from the Earth with the velocity necessary to overcome its gravitational attraction will trace an orbit relative to the Earth; since the latter circles the Sun, the rocket will simultaneously follow the Earth
11 and accomplish a path around the Sun, as its satellite.

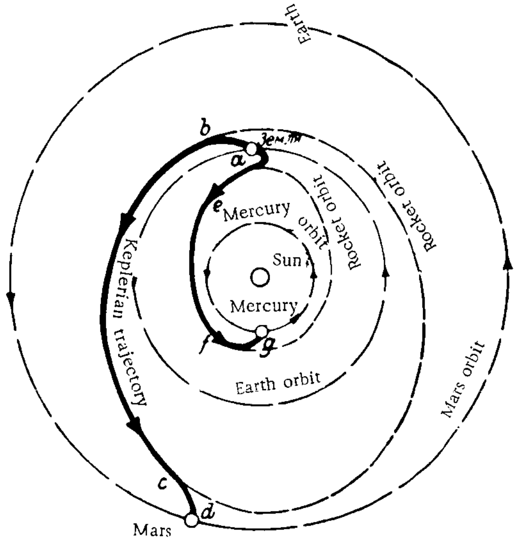
In order to escape the Earth's attraction and approach the Sun or, on the other hand, leave the solar system entirely, the velocity of the spacecraft must exceed the orbital velocity of the Earth which has an average value of 29.45 km/sec.

If the Earth were alone, immobile in space, a rocket launched from its surface would be influenced by no other gravitational field. Thus it would follow a correct elliptic orbit and return finally to the point of takeoff on the orbit. For high initial velocities, approaching the parabolic one, such a journey could take a very long time — even millions of years. Nevertheless, the velocity of the rocket on its return to the Earth would equal the launching velocity.

Suppose that a rocket leaves the Earth in a direction opposite to that of the Earth's motion around the Sun (Figure 11). Should its climb be stopped/at a given altitude, the craft would continue to circle the Sun with the velocity of 29.45 km/sec (same as the Earth's); but should it regain its motion in the initial direction of climb, its velocity relative to the Sun

would decrease and it would tend to approach the Sun. Thus, by choosing the correct velocity, it is possible to approach the orbit of any inner planet of the solar system, Mercury for example, and to circumnavigate it or even land on it at the right moment. This is the case shown in Figure 11, curve aefg. (The flight direction of the rocket will be as that of the Earth, counterclockwise.) At a certain velocity the rocket may even fall on the Sun. But if a rocket has to reach an outer planet, it must be given a velocity in the Earth-motion direction; thus its velocity relative to the Sun will exceed 29.45 km/sec and it will trace a Keplerian trajectory outside the terrestrial orbit. Analogous to the previous case, a rocket
12 can be directed to approach Mars, become its satellite or descend on it. This case is represented on Figure 11 by the curve abcd.

(11)



If a rocket is to leave the solar system and not return to it, its initial velocity must be $29.45\sqrt{2} \simeq 42$ km/sec (equation 4).

5. THE ATMOSPHERIC ENVELOPE

The precise law determining the air resistance in the course of a flight of a high-velocity rocket is unknown. It is difficult to derive such a law because it depends on many factors.

Very few laboratory experiments concerning air resistance at high velocities (equal to and higher than the velocity of sound = 337 m/sec) have been carried out. There are records on experiments performed in Germany (Göttingen) and the U.S.A. (Washington); in the latter case the air velocities extended from 0.5 to 1.08 times the velocity of sound. Since in artillery, however, numerous experiments have been performed to determine the air resistance of projectiles, one is obliged for the time being to use these data for calculations of the aerodynamic drag of a rocket.

Air resistance is influenced by the following factors:

1. Rocket diameter.
2. Rocket shape.
3. Position of its axis relative to the flight trajectory.
4. Air density.
5. Flight velocity. One should remember that the phenomena are different at flight velocities equal to or higher or lower than the velocity of sound.
6. Air skin friction with the rocket.
7. Head resistance due to the shock-wave phenomenon.
8. Suction at the rear part of the rocket.

Thus the air resistance of a rocket can be determined by the formula:

$$R = w_1 + w_2 + w_3, \tag{5}$$

where w_1 — skin friction resistance,
w_2 — head resistance,
w_3 — rear drag.

Sommerfeld simplifies this expression into:

$$R = w_1 + w_2, \tag{6}$$

where w_1 — skin friction resistance, w_2 — the remaining aerodynamic resistance. He defines w_1 as:

$$w_1 = F \cdot a \cdot v^2, \tag{7}$$

13 where F — cross-sectional frontal area of the rocket,
a — a coefficient of friction,
v — rocket velocity, m/sec.

He expresses the second component of the aerodynamic drag in the form

$$w_2 = F \cdot A \cdot \left(1 - \frac{s^2}{v^2}\right), \tag{8}$$

where A — a coefficient; s — velocity of sound, m/sec.

Thus, at

$$v = s,$$

we have

$$w_2 = 0.$$

The resistance per unit of the cross-sectional frontal area will be:

$$R = w_1 + w_2 = av^2 + A\left(1 - \frac{s^2}{v^2}\right).$$

Finally, relating it to a velocity of $v = 1$ m/sec, I obtain the so-called air-drag coefficient:

$$k = \frac{R}{v^2} = a + \frac{A}{v^2}\left(1 - \frac{s^2}{v^2}\right). \qquad (9)$$

If the values of a and A_1 are derived from experiments, then the air-drag coefficient for the velocity v will be determined from equation (9). Multiplying it by F and v^2 I will obtain — after Sommerfeld — the total air resistance to the rocket motion.

The atmospheric resistance is ofteh expressed in the form:

$$R = kF \cdot \frac{\delta}{\delta_0} \cdot i \cdot f(\alpha)\, f(v), \qquad (10)$$

where k — a coefficient,

F — cross-sectional frontal area of the rocket,

δ and δ_0 — air densities at altitude H and at sea level,

i — shape factor of the rocket,

$f(\alpha)$ — function depending on the position of the rocket axis relative to the flight trajectory,

$f(v)$ — function depending on flight velocity.

Oberth writes formula (10) in the form:

$$R = kF \cdot \frac{\delta}{\delta_0} v^2,$$

which gives $k = \frac{R}{F\frac{\delta}{\delta_0} v^2} = f(v)$. The corresponding variation of the function k is shown in Figure 12.

14 Figure 12 shows that the f(v) values somewhat decrease at the beginning (up to 200 m/sec), then sharply increase, reaching a maximum near 400 m/sec and from there on decrease more gradually but steadily.

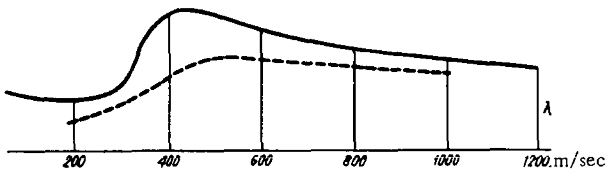


FIGURE 12. Change in air resistance after Oberth

The increase of the coefficient near the velocity of sound (330 m/sec) can be explained by: a) shock waves created at the rocket head [wave drag], b) creation of a vacuum at its rear. The decrease of the coefficient at supersonic velocities is explained as follows: the head resistance is nearly proportional to the square of the velocity (but is about 1.5 times higher than at low velocities), whereas the rear drag is constant and is equal to atmospheric pressure multiplied by the cross-sectional area.

Dividing the rear drag by the square of the ever-increasing velocity I obtain a continuous reduction of the rear drag coefficient.

If w' denotes head resistance and w'' the rear drag, then for increasing v the expression

$$k = \frac{w' + w''}{F \frac{\delta}{\delta_0} v^2}$$

goes to the limit

$$k_{lim} = \frac{w'}{F \frac{\delta}{\delta_0} v^2}.$$

In the case of a rocket the gas ejection reduces the suction at the rear and at high flight velocities it entirely disappears. The variation of the k-values is indicated in Figure 12 by the dashed line.

Goddard determines the aerodynamic drag by the following formula:

$$R = R_0 F \cdot \frac{\delta}{\delta_0}, \tag{11}$$

where R_0 is the atmospheric resistance related to a unit area of the rocket body cross-section. R_0 differs for different velocities.

Mallock's definition of the air drag is:

$$R_0 = 0.00006432\, v^2 \left(\frac{v'}{s}\right)^{0.375} + 480 \tag{11a}$$

(in the absolute system: foot, second, pound).

This formula is applicable to projectiles with tapered heads. Here:

v' — velocity of the body;

15 v — velocity of wave propagation in air just in front of the projectile: this velocity equals the projectile velocity in supersonic flight;

s — velocity of sound in undisturbed air.

480 (pounds) — a constant added at $v > 730$ m/sec (2,400 ft/sec) as a vacuum correction at the rear of the projectile.

Hohmann suggests the following relation to determine the air drag:

$$R = R_0 F \cdot i = \frac{\delta}{g} v^2 F \cdot i \tag{12}$$

where

v — flight velocity;

g — gravitational acceleration;

δ — specific weight of air;

R_0 — pressure exerted on 1 m^2 cross-sectional area perpendicular to the direction of flight;

F — area of the body cross-section perpendicular to flight direction;

i — shape factor of a body (for a thin plate $i = 1$; for a convex hemisphere i=0.5; for a cone with an apex angle φ $\left(\sin\varphi = \frac{9.35}{27}\right)$, i = 0.11, etc.).

Having determined the projectile climb velocities v at diverse altitudes h Hohmann presents the following table (Table 2) of v-values and computes expressions $\frac{\delta}{g} \cdot v^2$, which enter formula (12).

At altitudes exceeding 50 km and at velocities indicated in the table, the aerodynamic resistance may be neglected.

For altitudes under 50 km Hohmann takes an average value $\frac{\delta v^2}{g} = 12{,}000\ kg/m^2$.

TABLE 2

Altitude h, km	v^2, km^2/sec^2	$\frac{\delta v^2}{g}$, kg/m^2
0	0.00	0
1	0.04	4,600
2	0.08	8,000
3	0.122	11,000
4	0.162	13,000
5	0.202	14,200
6	0.243	15,100
8	0.323	15,500
10	0.404	15,200
15	0.606	13,000
20	0.810	8,500
30	1.214	3,440
40	1.620	1,200
50	2.028	370
60	2.434	110
80	3.250	7.5
100	4.070	0.4

16 Kranz formulates the law of atmospheric resistance as follows:

$$R = k \cdot r^2 \pi \frac{\delta}{\delta_0} \cdot i \cdot f(v)\ kg, \qquad (12a)$$

determining its value separately in each velocity zone.

The notations have the following meanings:

k — coefficient taken by some of the experimenters as dependent on projectile velocity (whereas others reject any dependence between the two),

r — radius of the body cross-section,

π — ratio between cross-section circumference and diameter,

δ and δ_0 — weights of 1 m^3 air in kg at the time of the experiment and at normal conditions, respectively,

i — shape factor,

f (v) — function dependent on the projectile flight velocity.

I will introduce some of the formulae suggested by different experimenters, which determine R for artillery shells.

1) N. Maievskii (Russia).

a) For a sphere, $i = 1$,

$$R = 0.012\, r^2 \pi \cdot \frac{\delta_i}{1.206} \cdot v^2 \left[1 + \left(\frac{v}{186}\right)^2\right], \text{ if } 376 \text{ m/sec} > v > 0.$$

$$R = 0.061\, r^2 \pi \frac{\delta_i}{1.206}\, v^2, \text{ if } 530 > v > 376 \text{ m/sec}.$$

b) For an oblong projectile with an ogival head and a radius of curvature 1 to 1.5 times the body diameter,

$$R = 0.012\, r^2 \pi \frac{\delta}{1.201}\, v^2 \left[1 + \left(\frac{\delta}{488}\right)^2\right], \text{ if } 280 > v > 0;$$

$$R = \overset{(11)}{0.026}\, r^2 \pi \frac{\delta}{1.206}\, v^6, \text{ if } 360 > v > 280;$$

$$R = 0.044\, r^2 \pi \frac{\delta}{1.206}\, v^2, \text{ if } 510 > v > 360 \text{ m/sec}.$$

2) Hajel (Holland).

For $i = 1$ for oblong projectiles with ogival heads with radius of curvature (r) equaling twice the body diameter:

$$R = \frac{(2r)^2 \cdot 1000 \cdot \delta \cdot i}{9.81 \cdot 1.201} \cdot a \cdot v^n,$$

where for v = 140 to 300 m/sec	$a = \overset{(6)}{0.084535}$;	$n = 2.5$
300 to 350 m/sec	$a = \overset{(11)}{0.05423}$	$n = 5$
350 to 400 m/sec	$a = \overset{(8)}{0.051381}$	$n = 3.83$
400 to 500 m/sec	$a = \overset{(4)}{0.07483}$	$n = 1.77$
500 to 700 m/sec	$a = \overset{(3)}{0.05467}$	$n = 1.91$

17 3) Maievskii-Zabudskii (Russia).

$i = 1$ (as indicated in case 2)

$$R = a \cdot r^2 \pi \frac{\delta \cdot i}{1.206} v^n$$ where for v =

v		a	n
0 to 240	m/sec.	0.0140	2
240 to 295	m/sec	$\overset{(4)}{0.05834}$	3
295 to 375	m/sec	$\overset{(9)}{0.06709}$	5
375 to 419	m/sec	$\overset{(4)}{0.09404}$	3
419 to 550	m/sec	0.0394	2
550 to 800	m/sec	0.2616	1.7
800 to 1000	m/sec	0.7131	1.55

4) Chapel-Vallier-Scheve.

$i = 1$, as in case 2. (Apex angle of the ogival projectile head is $83° = 2\gamma$.)

for $v > 330$ m/sec, $R = \dfrac{r^2 \cdot 10{,}000 \cdot \delta \cdot i}{9.81 \cdot 1.206} \cdot 0.125\,(v - 263)$;

(6)

for $v < 300$ m/sec, $R = \dfrac{r^2 \cdot 10{,}000 \cdot \delta \cdot i}{9.81 \cdot 1.206} \cdot 0.03814 \cdot v^{2.5}$;

at $v < 330$ m/sec, $i = 1$;

at $v \geqslant 330$, $i = \dfrac{\gamma\left[v - (180° + 2\gamma)\right]}{41.5\,(v - 263)}$.

In case the projectile configuration differs from that mentioned above, then:

at $v < 330$ m/sec:	$i = 0.67$	0.72	0.78	1.10
	for $\gamma = 31°$	33.6°	36.9°	48.2°

5) Siacci.

$R = 338 \cdot r^2 \cdot \delta \cdot i \cdot f(v)$ kg,

where r — projectile body radius in m,
δ — weight of 1 m^3 of air in kg,

$$f(v) = 0.2002 \cdot v - 48.05 + \sqrt{(0.1648 \cdot v - 47.95)^2 + 9.6 + \frac{0.0442 \cdot v\,(v - 300)}{371 + \left(\frac{v}{200}\right)^{10}}}.$$

$i = 0.896$ for an ogival head with a radius of curvature equaling $4r$ and a head height of $2.6r$. If the head-height range is from $1.8r$ to $2.2r$ then $i = 1$.

18 Figure 13 shows the effect of velocity on R. For convenience of representation the expression $\frac{10^6}{v^2} \cdot f(v)$ is used instead of the function $f(v)$ itself.

The curve has an inflection point at $v = 340$ m/sec and a maximum at $v = 500$ m/sec. For some of the velocities the $f(v)$-values are written on the curve itself.

19 I will study more closely a cylindrical projectile with an ogival head. Its body diameter is $2r$, the radius of curvature of the head is $r_1 = rn$, and the head height is h. The angle between the tangent to the contour of the head at its apex and the projectile axis is λ.

In Table 3 are given the ratios of the head dimensions for different projectiles and shape factors i (for ogival and conical heads). Here i is defined as the ratio of resistance of a cylindric projectile with a given head to the resistance of an identical cylinder with a flat frontal surface (perpendicular to the axis).

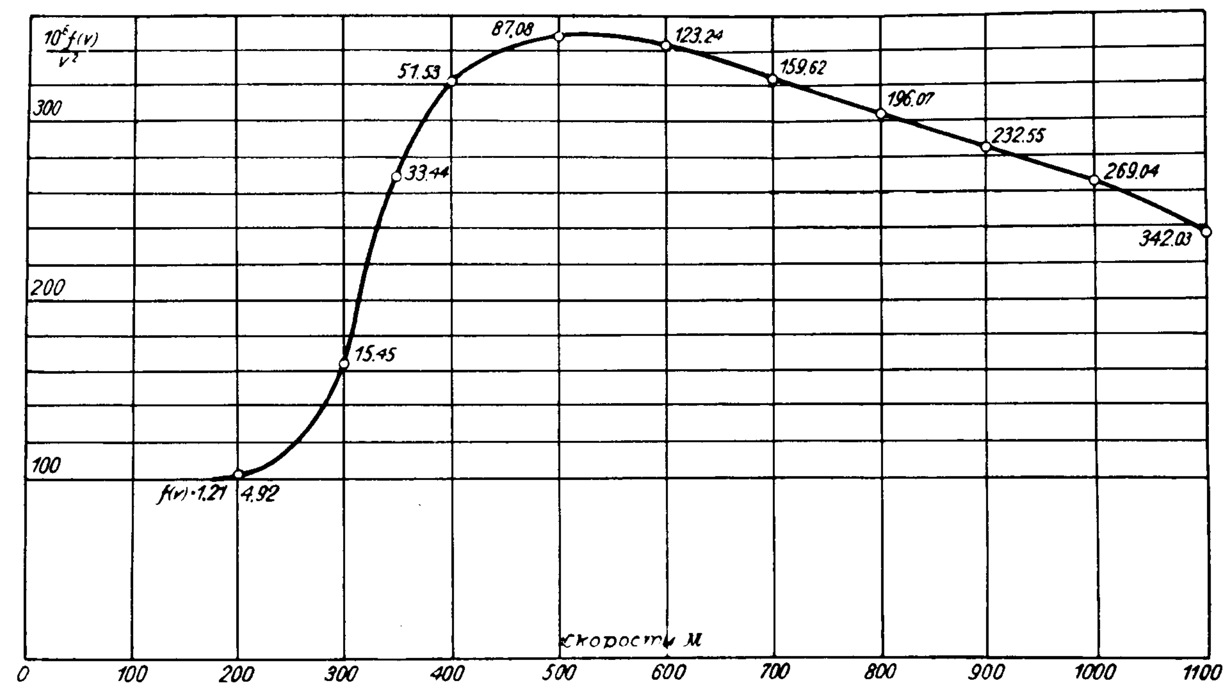


FIGURE 13. Curve of aerodynamic resistance

TABLE 3

$\frac{r_1}{2r}$			0.5	1	1.5	2	2.5	3
$\frac{h}{2r}$			0.5	0.866	1.118	1.323		1.638
γ			90° hemisphere	60°	48°13'	41°25		30°34
$\cos\gamma$			0	1/2	2/3	3/4		5/6
Coefficients	ogival heads	after Lesley	0.666	0.504	0.419	0.366	0.331	
		after Duchenne	0.858	0.752	0.675	0.617	0.571	
		after Newton	0.500	0.292	0.204	0.156	0.127	
	conical heads	after Lesley	0.707	0.500	0.409	0.353	0.317	
		after Duchenne	0.943	0.800	0.663	0.628	0.575	
		after Newton	0.500	0.250	0.167	0.125	0.100	

Air temperature and density

Air temperature decreases with altitude. According to observations, however, its decrease ceases at an altitude of about 12 km, and further, up to 30 km, it seems to stay constant (-60°C).

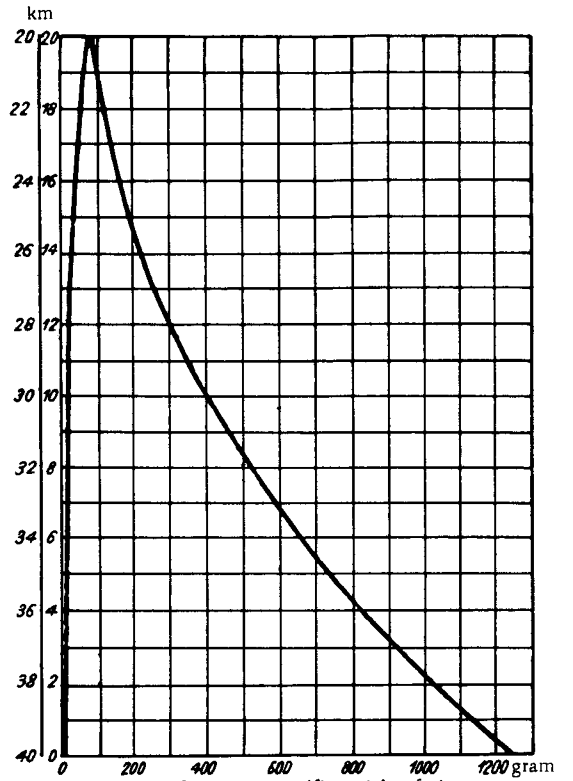
Remark. The lowest observed temperature in the atmosphere near the Earth's surface was -86°C.

20 Air density also decreases with altitude; a series of formulae were suggested and many observations carried out to determine it. For convenience in aerodynamic calculations, characteristic variations of atmospheric properties versus altitude have been prepared (based on experimental measurement data obtained by airplanes and balloons). So-called standard atmospheres have been worked out by the Russians, the French, the Germans and the Americans.

The curves of Figure 14 show the changes in weight of 1 m^3 of air in grams or air-to-water density ratios. The right curve presents weights at altitudes from zero to 20 km and the left one from 20 to 40 km.

Hohmann assumes in his calculations the following variation with altitude of the specific weight of air for spaceflights (Table 4).

Tsiolkovskii supposes that the density of matter in space is $1:16 \cdot 10^{18}$ of the air density.



Annual-average specific weight of air
(the weight of 1 m^3 in grams)

FIGURE 14. Change in weight of air with altitude

TABLE 4

Altitude h, km	Weight of 1 m^3 air, kg	Altitude h, km	Weight of 1 m^3 air, kg
0	1.3	50	0.00187
1	1.15	55	0.00915
2	1.00	60	0.000448
3	0.90	65	0.000217
4	0.80	70	0.0001025
5	0.70	75	0.0000497
6	0.62	80	0.0000230
8	0.48	85	0.0000106
10	0.375	90	0.0000049
15	0.215	95	0.0000022
20	0.105	100	0.00000098
25	0.055	105	0.000000423
30	0.0283	110	0.000000185
35	0.01464	150	0.00000000003
40	0.0074	200	0.0000000000000023
45	0.00376	400	0.00 ....

21 6. PROPELLANT

a) General properties

One of the most important questions posed in rocketry concerns the propellant, whose combustion products burst out from the nozzle of the rocket and propel the latter by the reaction of the gas jet. Therefore it is essential to know which types of propellant release the largest energy quantities and what will be the ejection velocities of the gaseous products of the explosion. Energy is characterized by the calorific value liberated by 1 kg of fuel and expressed in kilocalories (1 kcal = 427 kg · m). Diverse propellants release a certain amount of work during the combustion process (i. e., reaction with oxygen) and can be classified according to the following criteria:

I. Rate of combustion.
  A. Slow-burning: wood, coal.
  B. Fast-burning, used in internal combustion engines: gasoline, kerosene, alcohol, etc.
  C. Low-explosive: powders.
  D. High-explosive: dynamite, pyroxylin, etc.

II. Oxygen content.
  E. Oxygen supplied externally: gasoline, wood, coal.
  F. Containing oxygen: powder, dynamite, etc.

22 III. State.
  G. Gaseous: illuminating gas, hydrogen, etc.
  H. Liquid: gasoline, alcohol, etc.
  I. Gelatinous: nitrogelatin, etc.
  J. Powders: gun powder.
  K. Solid: compressed gun powder, coal, wood.

IV. Purpose.
  L. To obtain heat: wood, coal.
  M. To obtain work:
    a) In engines.
    b) In weapons (rifles, guns, bombs).
    c) In mining (explosions).
    d) To explode other substances (mercury fulminate).

If a propellant containing no oxygen burns in an oxygen-rich atmosphere, a unit weight of propellant, e. g., 1 kg, will release much more heat energy than the same quantity (1 kg) of a prepared mixture of that propellant with oxygen.

Tables 5 and 6 comprise the calorific data (in kcal) and the theoretical ejection velocities (in m/sec) of 1 kg-quantities of oxygen-bearing propellants (A) and oxygen-free (B) substances.

A burst produces a certain volume of gases; the larger this volume is, the more work it can do. The gas particles produced during the combustion of 1 g of the original propellant will occupy a larger volume if their
23 molecular weight is lower. One cubic centimeter of any gas contains a constant number of molecules at equal conditions of temperature and pressure. Hydrogen molecules are the lightest; 1 g of liquid hydrogen

converted into gas occupies a volume of 11.2 liters (at 0°). So far no other substance has been found from which a larger gas volume can be obtained. The volume of gases increases with the temperature of explosion, but much less than during the conversion of a solid or liquid substance into its gaseous state. Raising the explosion temperature by a few thousand degrees will increase the effectiveness 10 to 15 times. This temperature depends on how much heat is released in the course of the explosion; thus to obtain a higher efficiency of the process one should choose fuel substances which will liberate more heat at optimum conditions of the chemical reaction. However, it seems pointless to expect further substantial successes in developing high-explosive fuels. The existing compounds already produce much heat and very high temperatures: gun powder — 2,380°, pyroxylin — 3,100°, detonating gas — 3,750°, nitroglycerine — 4,250°, mercury fulminate — 4,350°, sheddite — 4,500°.*

Because of the quick release of their chemical energy explosives possess great power. The action of 1 kg dynamite is equivalent for example to the effect of an engine of 20–30 million h. p., functioning during the same time (1 kg dynamite burns out in 0.00002 sec).

(22) TABLE 5

	Substance	Product	Calorific value, kcal	Ejection velocity, m/sec
A. Oxygen-bearing. Numerical values for 1 kg of substance	Hydrogen + oxygen	Water vapor	3,220	5,180
		Water	3,736	5,600
		Ice	3,816	5,650
	Carbon + oxygen	Carbon dioxide	2,210	4,290
	Gasoline + oxygen	Water + carbon dioxide	2,370	4,450
B. Oxygen supplied externally. Numerical values for 1 kg of substance	Hydrogen	Water	34,462	
	Hydrogen	Water vapor	29,000	
	Carbon	Carbon dioxide	8,100	
	Hydrocarbons	Carbon dioxide + water	10,000–13,000	

* Certain metals burned in an atmosphere of ozone render very high explosion temperatures as well: lithium (LiOH – 5,100°), boron (B_2O_3 – 5,000°), aluminum (Al_2O_3 – 4, 100°), etc. (The formulae express the composition of the ejected products.)

TABLE 6

Fuel	Weight of 1 m^3, kg	Calorific value of 1 kg, kcal	Ejection velocity of gases, m/sec	
Wood	310–1,390	2,400–4,730		Oxygen-free
Bituminous coal	800	6,500–8,470		
Pure alcohol	795	6,700		
Petroleum	810–940	10,000–11,000		
Kerosene	770–860	10,500		
Powder for ship's rocket	–	528	314	Oxygen-bearing
Powder for an ordinary rocket	–	545	292	
Smoky powder producing 290 liters of gas	1,200	165–720	2,893	
Gun powder "Du Pont No. 3" in a steel rocket	–	972	1,907–2,290	
Pyroxylin	1,300	1,025	2,900	
Powder "Infallible" in a steel rocket	–	1,278	2,082–2,434	
Gelatinous dynamite	1,660	1,293	–	
Nitrogelatin	1,630	1,550	–	
Smokeless powder	705–840	1,000–1,570	–	
Nitroglycerine	1,600	1,580	2,900–3,500	

24 TABLE 7

No.	Object	Conditions	Velocity, km/sec
1	Chlorine molecule	At 0°C and 760 mm in Crookes' cathode tube	0.24
2	Carbon-anhydride molecule	The same	0.36
3	Falling body	From an altitude of 20 km to the Earth's surface, in vacuum	0.60
4	Hydrogen molecule	As in 1 and 2; velocity increases in proportion to square root of absolute temperature.	1.8
5	Hydrogen molecule	In the hydrogen flame arc	7.04
6	Falling body	From infinity to the Earth's atmosphere	11.5
7	Meteorite	At an altitude up to 160 km	20
8	Electron	In vacuum tube at potential difference of 0.4 V	100
9	Mercury atom	The same, at potential difference of 10,000 V	100
10	Hydrogen molecule	On stars at 2,730,000°C	269
11	Electron	In vacuum tube at potential difference of 1 V	295
12	Falling body	From infinity to the Sun	600
13	Electron	In vacuum tube at potential difference of 5.5 V	1,400
14	Hydrogen atom	In vacuum tube at potential difference of 10,000 V	1,400
15	Electron	In vacuum tube at potential difference of 25 V	3,000
16	Electron	In vacuum tube at potential difference of 100 V	5,950
17	Helium	–	$(10–30) \cdot 10^3$
18	Radium	–	$30 \cdot 10^3$
19	Electron	In vacuum tube at potential difference of 100,000 V	$58.3 \cdot 10^3$
20	α-rays	–	$300 \cdot 10^3$

It would seem that to obtain greater power one should look for explosives with a higher rate of dissociation. However, many of the existing substances are dissociated almost instantaneously (pyroxylin, mercury fulminate).

Disintegration of radioactive elements (which is not a chemical process but a splitting of an atom nucleus) could serve as an alternative and much more powerful energy source — tens of thousands times more than the conventional explosives. If radium (1 kg of which possesses a calorific value of $3.7 \cdot 10^9$ kcal) or uranium or any other radioactive element could be forced to disintegrate as quickly as dynamite, explosives of outstanding power could be obtained. So far, however, atomic energy is unattainable. It is true that sometimes radioactive elements, e. g., radium, disintegrate spontaneously, but this is a slow process. One gram of radium decays by half in 1,750 years. Rutherford succeeded in splitting nitrogen atoms, but that was achieved in a very complicated process which demanded enormous quantities of energy.

Velocities of some objects are given in Table 7.

25 b) Computation of propellant energy

Consider an example of energy release computation for a most effective propellant, a mixture of hydrogen (H) and carbon (C).

Burning means combination of the propellant with oxygen.

Burning hydrogen can produce water (H_2O) or water vapor, whereas the products of carbon combustion may be carbon dioxide (CO_2) or carbon monoxide (CO).

Combination of 1 kg hydrogen with oxygen at $t° = 0°C$ to produce liquid water yields 34,462 kcal. The heat release for the same reaction yielding water vapor creates $600 \cdot 9 = 5{,}400$ kcal less, i. e., 29,000 kcal. The difference (5,400 kcal) is the heat of transformation of water to vapor. Notice that burning of 1 kg hydrogen produces 9 kg of steam.

The heat of formation of carbon dioxide from 1 kg of carbon is 8,100 kcal.

Chemical reactions which take place during combustion are accomplished in definite weight proportions, corresponding to the atomic weights of the combustible elements.

Atomic weights: hydrogen (H) — 1, carbon (C) — 12, oxygen (O) — 16.

Water is obtained when two hydrogen atoms combine with one oxygen atom. One carbon atom burns with two oxygen atoms into carbon dioxide, with only one into carbon monoxide. Here are the formulae expressing the above statements:

$$H_2 + O = H_2O; \quad C + O_2 = CO_2; \quad C + O = CO.$$

Thus burning of 1 kg hydrogen necessitates $\frac{16 \cdot 1}{2} = 8$ times more oxygen, or 8 kg; burning of 1 kg carbon into carbon dioxide (CO_2) demands $1 \cdot \frac{16 \cdot 2}{12} = 2.66$ times more oxygen or 2.66 kg. But if carbon burns into carbon monoxide (CO), then $\frac{16 \cdot 1}{12} = 1.33$ kg oxygen is needed versus 1 kg carbon.

In the case of an oxygen-bearing propellant containing mainly hydrogen, the heat released will be only $\frac{1}{1+8}=\frac{1}{9}$ of the calorific value of pure hydrogen. For example, $\frac{29,000}{9}=3,220$ kcal for hydrogen-oxygen mixture and $\frac{8,100}{1+2.66}=2,210$ kcal for carbon-oxygen mixture.

It would be advantageous to use pure hydrogen because of its high calorific value. However, several hydrocarbons, e. g., benzene (C_6H_6), pentane (C_5H_{12}), heptane (C_7H_{16}), methane (CH_4), and others of the form C_mH_n also possess quite a high efficiency.

Let us determine the calorific value of such a mixture (C_mH_n). One kg of it incorporates 12 m + 1 n parts by weight and the fraction of hydrogen.

$$\frac{n}{12m+n}$$

26 whereas that of carbon is

$$\frac{12m}{12m+n}.$$

Hydrogen calorific value will be

$$\frac{n}{12m+n}\cdot 2,900$$

and that of carbon:

$$\frac{12m}{12m+n}\cdot 8,100 .$$

The total calorific value of 1 kg of the mixture will be expressed in the following form:

$$\frac{29,000n+97,200m}{12m+n}$$

and the amount of oxygen needed for the burning will be

$$\frac{16\cdot m\cdot 2+16\cdot n\cdot\frac{1}{2}}{12m+n}=\frac{32m+8n}{12m+n}\text{ kg oxygen.}$$

The equation of the chemical reaction will take the form:

$$C_mH_n+\left(m+\frac{n}{4}\right)O_2=mCO_2+\frac{n}{2}H_2O.$$

But if C_mH_n also includes some extra oxygen, the following quantities should be used with 1 kg of this mixture:

$16\,m \cdot 2$ for the combustion of carbon

$16\,n \cdot \frac{1}{2}$ for the combustion of hydrogen

and altogether 1 kg contains the following number of parts by weight:

$$12\,m + n + 16\,m \cdot 2 + 16 \cdot n\,\frac{1}{2} = 44\,m + 9\,n.$$

The expression for the overall calorific value of 1 kg of the mixture will be:

$$\frac{29{,}000\,n + 97{,}200\,m}{44\,m + 9\,n}. \qquad \text{(a)}$$

1 kg of this mixture should comprise

$$\frac{32\,m + 8\,n}{44\,m + 9\,n} \text{ kg oxygen.}$$

I will obtain the following quantities of energy created during the combustion of different propellants mentioned above (1 kg of mixtures with oxygen).

Ejection velocity

27 Pure carbon (C):

Formula (a): $n = 0$; $G = \frac{97{,}200}{44} = 2{,}210$ kcal, or

$2{,}210 \cdot 427 = 945{,}000$ kg · m . . . . 3,500 m/sec

Benzene (C_6H_6) $G = \frac{97{,}200 \cdot 6 + 29{,}000 \cdot 6}{44 \cdot 6 + 9 \cdot 6} = 2{,}380$ kcal

or 1,016,000 kg · m . . . . 3,640 m/sec

Heptane (C_7H_{16}) $G = \frac{97{,}200 \cdot 7 + 29{,}000 \cdot 16}{44 \cdot 7 + 9 \cdot 16} = 2{,}534$ kcal

or 1,080,000 kg · m . . . . 3,650 m/sec

Pentane (C_5H_{12}) $G = \frac{97{,}200 \cdot 5 + 29{,}000 \cdot 12}{44 \cdot 5 + 9 \cdot 12} = 2{,}550$ kcal

or 1,090,000 kg · m . . . . 3,750 m/sec

Methane (CH_4) $G = \frac{97{,}200 + 29{,}000 \cdot 4}{44 + 9 \cdot 4} = 2{,}665$ kcal

or 1,140,000 kg · m . . . . 3,840 m/sec

Pure hydrogen (H) $G = \frac{29{,}000}{9} = 3{,}220$ kcal, or

1,375,000 kg · m . . . . 4,120 m/sec

The ejection velocity of gases depends on the magnitude of this energy and is derived as follows:

$$E = \frac{1}{2} mv^2,$$

or

$$G = \frac{1}{2} \cdot \frac{1}{g} \cdot v^2;$$

i. e.

$$v = \sqrt{2g \cdot G \cdot 427} = 91.5\sqrt{G}.$$

The high temperature of the combustion process results in a certain loss of energy due to dissociation. At low pressures about $^1/_3$ of the energy is lost, whereas at high pressures this loss is insignificant. Therefore, the ejection velocity formula should be corrected to

$$v = 73\sqrt{G}.$$

The ejection velocities thus obtained for the mentioned mixtures are presented above.

Remark: for atomic hydrogen the velocity is taken as v = 12,000 m/sec.

28 Oberth computes the jet velocity employing Zeiner's formula:

$$c = \sqrt{2 \cdot 9.81 \cdot \frac{K_0}{K_0 - 1} P_0 \cdot V_0 \left(1 - \frac{P}{P_0}\right)^{\frac{K_0 - 1}{K_0}}}$$

where K_0 — ratio $\frac{\text{specific heat of gas at constant pressure}}{\text{specific heat of gas at constant volume}}$,

P_0 — absolute pressure in the combustion chamber, kg/m^2,

P — absolute pressure in the investigated place in the nozzle, kg/m^2,

V_0 — gas volume, m^3.

Oberth takes for the second-stage alcohol rocket $K_0 = 1.30$. For the upper stage, where hydrogen and water vapor explode with oxygen, he uses a different value of K_0 (which depends on the oxygen-hydrogen weight ratio). Variation of this ratio from 0.8 to 1.9 changes K_0 correspondingly from 1.4 to 1.383. For oxygen $K_0 = 1.406$.

The lowest velocity is c = 1,530 m/sec. In his calculations Oberth assumes c = 3,000 m/sec.*

An example of a rocket engine computation. I will use the data already introduced to evaluate the performances of a reactive device patented by A. A. Andreev (recorded 18 February 1921). His idea is: a knapsack containing liquid methane and oxygen propellant is fixed to the back of a man. Igniting the mixture (after its conversion to a gaseous state) and forcing the products of combustion through two rocket nozzles, one should be able, according to the author of the design, to perform air jumps of 22 km in about seven minutes (200 km/hour).**

* Regarding the theory of gaseous ejection refer also to the article by K. Baetz "Der Raketenschuss und der zweite Hauptsatz der Wärmetheorie." — Die Rakete, p.89, 1928.

** Details about rocket propellants can be found in the article by F. Hoeftt "Betriebstoffe der Raumschiffe" in the book: "Die Möglichkeiten der Weltraumfahrt," p.153, Leipzig, 1928.

The liquid substances in the knapsack were to be kept in fire-resistant containers (kaolin and refractory clay), which can withstand temperatures up to 2,000°.

Data for the computation:

Weight of the liquid fuel (methane and oxygen)	8 kg
Weight of knapsack	24 kg
Weight of man	56 kg
Lift margin	12 kg
Total	96 kg

Assuming complete combustion, the calorific volume of the 8 kg of propellant mixture will be:

$$8 \times 2,665 \times 427 = 9,103,640 \text{ kg} \cdot \text{m}.$$

29 Assuming 34% losses the energy obtained will be:

$$Q = 0.66 \cdot 9,103,640 = 6,008,400 \text{ kg} \cdot \text{m}.$$

The work necessary to transfer 96 kg a distance of 22 km equals:

$$Q_1 = 96 \cdot 22,000 = 2,112,000 \text{ kg} \cdot \text{m}.$$

Reserve

$$\frac{Q}{Q_1} = \frac{6,008,400}{2,112,000} = \sim 3.$$

Assuming the specific weights of liquid methane and oxygen to be 0.75 and 1.4 respectively, 8 kg of the liquid propellant will occupy the following volume:

liquid methane $\frac{1,600}{0.35} = 4,570 \text{ cm}^3$,

liquid oxygen $\frac{6,400}{1.4} = 4,570 \text{ cm}^3$.

In the gaseous state 1 liter of oxygen weighs 1.429 g and 1 liter of methane 0.716 g. Due to this information I find that these substances, being gaseous under normal pressure, occupy the following volumes:

oxygen $\frac{6,400}{1,429} = \sim 4,500$ liters;

methane $\frac{1,600}{0.711} = \sim 2,235$ liters.

The total is 6,735 liters = 6,735,000 cm^3.

The volume of the combustion products exceeds 15 times the initial volume of the propellant, i. e., 6,735,000 · 15 = 101,025,000 cm^3. Supposing the total area of both exit nozzles to be 8 cm^2 and the pressure (for a conical nozzle) 17.76 kg/cm^2, the reaction obtained is $17.76 \cdot 8 = 142.08 > 96$ kg.

Taking the jet velocity as 300 m/sec = 30,000 cm/sec, I can compute the burning time

$$\frac{101{,}025{,}000}{300 \cdot 100 \cdot 8} = \sim 420 \text{ sec} = 7 \text{ min.}$$

This calculation provides only a general notion about the performance of the device and must be verified by experiments. Notice that the author assumes 300 m/sec for the jet velocity, whereas methane itself gives 3,840 m/sec (almost 13 times more). Thus the flight will take $\frac{7}{13}$ minutes instead of seven minutes. But the accelerations created during the functioning of the engine of this knapsack-rocket device turn out to be dangerous for the human being. The design also does not suggest any means of velocity control.*

30 7. PHYSIOLOGICAL EFFECTS OF ACCELERATION

In the course of interplanetary flights high accelerations and decelerations will be encountered. The question is what are the upper limits of acceleration or deceleration a man can withstand without damaging his body, and for how long. Also to be investigated is the way a man will feel during a prolonged flight without acceleration or deceleration, i. e., at constant velocity in a medium without gravity. Finally, means of minimizing all these dangerous effects must be found.

I will treat the subject in the following order: a) acceleration-deceleration effects and how to reduce their influence; b) influence of the absence of acceleration and deceleration on the human body and means of reducing these effects.

a) Acceleration and deceleration effects

The weight of a body is determined from its pressure on a certain support (a balance plate, for example). This pressure is proportional to the product of the mass of a body and its acceleration. The acceleration acting on a body on the Earth's surface is the gravitational one and has values of g between 9.80 and 9.83 m/sec^2.

The terms "acceleration" and "deceleration" stand for velocity increase and decrease, correspondingly. Both represent the same physical effect of velocity gradient and will be referred to, simply, as "acceleration effects."

According to the law of relative motion these effects will take place: 1) when the body is at rest but its molecules are subjected to acceleration

* Andreev's design (with the corresponding drawing) is described in N. Rynin "Rakety" (Rockets), p. 76. Leningrad, 1929. [English translation by IPST, TT 70-50114, NASA TT F-643.]

(e. g., pressure of a weight on a balance plate) and 2) when the body accelerates but its molecules stay at rest (e. g., inertia effect on the passengers of a tram under high accelerations). The effect of acceleration is measured in the same units as acceleration itself, i. e., in m/sec^2.

A pilot pressed into his seat at the highest point of a vertical loop or bicycle wheels remaining attached to the circumference of the "devil's wheel" at its highest point are good examples of the acceleration effect. Accelerations appear under conditions of velocity changes during a rectilinear motion or while moving along a curvilinear path. The following formulae apply to a uniformly accelerated rectilinear motion of a body:

$$v = at; \quad s = \frac{1}{2} at^2; \quad t = \frac{v}{a}; \quad s = \frac{1}{2}\frac{v^2}{a}; \quad a = \frac{v^2}{2s},$$

where: v — velocity, m/sec,
t — time, sec,
a — acceleration, m/sec^2,
s — path, m.

31 The centrifugal acceleration of a body moving uniformly along a circular path is

$$b = \frac{v^2}{r} \ m/sec^2,$$

where: v — circular velocity, m/sec,
r — radius of the circle, m.

It has already been proved at various times that men can withstand the effects of acceleration without any damage to their health.

1. In war a pilot flying at 60 m/sec performed four successive spiral turns of 140 m diameter, thus experiencing during 29 seconds an acceleration of approximately 5.15 g $\left(b = \frac{60^2}{70} = 51.5 \ m/sec^2 = \sim 5.15 \ g\right)$ with no negative effects on his health.

2. In Italy experiments were performed with catapulting manned aircraft. The catapult length was 15 m and the launching velocity at its end after about 1 sec was 28 m/sec. Thus accelerations up to 31.7 m/sec^2 = 3.17 g were obtained with no harm to the pilot.

3. Similar experiments were carried out in France, near Brest, which did not affect the pilot at all. The catapult length was 20.25 m, the takeoff run was 13 m, the velocity at the catapult end = 22 m/sec and the acceleration about 2 g.

4. A fireman jumped from a height of 25 m. The moment he touched the canvas his body was in a horizontal position and he went down with it another meter — with no damage to his health. When he touched the taut canvas his velocity was $v = \sqrt{2g \cdot 25}$ and the subsequent deceleration $a = \frac{2g \cdot 25}{2 \cdot 1} = \sim 240 \ m/sec^2 = \sim 24 \ g$.

5. A swimmer in a standing position jumping from a height of 8 m submerged 2 m, i. e., an acceleration of 40 m/sec^2, without any ill effects.

6. Another swimmer jumped backwards from a 2-m height and collided with the water surface with his body in an almost horizontal position (sliding a little on it). Without being injured his body experienced the following accelerations: the back skin – 200 m/sec^2, the back muscles and the kidneys – 160 m/sec^2, other parts of the body – 80 m/sec^2, the head and the bones – 70 m/sec^2.

7. In the course of experiments carried out during aerobatic flights in the U. S. A. a pilot absorbed for short periods accelerations up to 7.8 g, with no damage.

A **human projectile** was the name of the flight from a gun demonstrated by the Italian Hugo Zacchini in December 1927 in the Leningrad circus.

The gun length was about 5 m and its inner diameter about 0.6 m. Before firing, the piston was placed in its extreme position in the gun
32 barrel. Air pressure was built up in a chamber behind the piston and Zacchini stood inside the barrel of the head. The moment the piston was released, Zacchini was pushed out of the gun, traced a parabolic trajectory and landed on a net. In the course of this accomplishment, witnessed by the present author, he reached a maximum height of 6 m traversing a horizontal distance of about 10 m in 1 to 1.5 sec, which corresponds to an acceleration not exceeding 2 g. According to the manager, flights to a height of 50 m are possible, space permitting. Moreover, Zacchini intends to reach a 150 m height in the future, which is hardly feasible. I will prove this with a simple calculation:

Denoting the maximum height by h m, the piston stroke by l, the gravitational acceleration by g and the acceleration of the motion inside the gun barrel by b, I obtain

$$v^2 = 2gh = 2bl; \text{ or } b = \frac{gh}{l}.$$

The assumption $b = 5g$, which is the acceleration safety limit, renders $l = \frac{h}{5}$. And for $h = 150$ m the piston stroke must be 30 m.

Putting $b = 10g$ (which is difficult to achieve as yet) gives

$$l = \frac{h}{10} = 15 \text{ m}.$$

Thus for $b = 5g$ and $l = 5$ m, I get $h = 25$ m. In reality this height will be somehow reduced by the air resistance.

Leinert's flights from a gun. In principle, the flights performed by Leinert in 1927 in Germany are the same as Zacchini's. Here are some specific data on his experiment: gunbarrel length 8 m, its inclination to the horizon 70°, piston stroke 6 m, height 25 m.

I will determine the acceleration effect during the launch and the descent.

33 During the launch the height of climb will be (Figure 15)

$$y = \frac{v_0^2}{2g} \sin^2\alpha,$$

where v_0 – initial velocity,
α – angle between the gun barrel and the horizontal,
g – gravitational acceleration.

In this case α and y are known. Therefore

$$v_0 = \sqrt{\frac{2gy}{\sin^2\alpha}}$$

The free flight begins from a 6 m-height. Thus $y = 25 - 6 = 19$ m; $\alpha = 70°$; $\sin\alpha = 0.940$; $\sin^2\alpha = 0.883$; $g = 9.81$ m. Therefore,

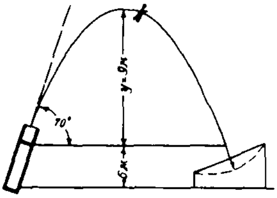


FIGURE 15. Flight of a man catapulted from a gun

$$v_0 = \sqrt{\frac{2 \cdot 9.81 \cdot 19}{0.833}} = 21.15 \text{ m/sec.}$$

This velocity is developed along a path of $s = 8 - 2 = 6$ m and the acceleration will be:

$$b = \frac{v^2}{2s} = \frac{21.15^2}{2 \cdot 6} = 37.3 \text{ m/sec}^2,$$

i. e., the overload or acceleration effect is almost 3.8 times higher than at normal conditions. However the flyer was not injured at all.

The overload while touching the net (stretched at a height of 6 m) with which he went an extra one and one-half meters was much higher. At such a braking distance the deceleration equaled:

$$b = \frac{v^2}{2s} = \frac{21.15^2}{2 \cdot 1.5} = 149 \text{ m/sec}^2;$$

or more than 15.2 times the normal acceleration.

The duration of the free-flight phase:

$$T = \frac{2v_0\sin\alpha}{g} = \frac{2 \cdot 21.15 \cdot 0.940}{9.81} = 4 \text{ sec.}$$

Throughout all his flights Leinert experienced no physiological problems.

Notice that at the launch the body axis vector was pointed in the flight direction, whereas during the descent it was perpendicular to its trajectory. Should such a flight be performed by means of a rocket, its weight would be determined as follows.

The acceleration in the gun barrel was 37.3 m/sec^2, so the rocket acceleration should be about 47 m/sec^2.

Denoting the rocket reaction thrust by P and its total weight (rocket engines + payload) by G, I will obtain the relation

$$b = \frac{v}{t} = \frac{P}{G} \cdot g = 47 \text{ m/sec}^2.$$

34 A rocket producing a thrust of 4.8 kg during a burning time of 0.15 sec will lift (based on experiments) a load:

$$G = \frac{4.8 \cdot 9.81}{47} = 1 \text{ kg}$$

at a climbing speed

$$v = \frac{P}{G} \cdot g \cdot t = \frac{4.8}{1.0} \cdot 9.81 \cdot 0.15 = 7.05 \text{ m/sec.}$$

Relative to the Earth the velocity will be:

$$v' = \left(\frac{P}{G} - 1\right) gt = 3.8 \cdot 9.81 \cdot 0.15 = 5.6 \text{ m/sec}.$$

The weight of the rocket which provides this effect is 55 gram, thus allowing $1 - 0.055 = 0.945$ kg for the payload. But to accelerate this payload to a velocity $V = 21.15$ m/sec one must use a large number of rockets.

Denoting the weight of a single rocket by G', the payload lifted by it by N and the full payload $L = 100$ kg, I will derive the expression for the necessary number of rockets:

$$n = \frac{L}{G'}\left[\left(1 + \frac{G'}{N}\right)^{\frac{v}{v'}} - 1\right] = \frac{100}{0.055}\left[\left(1 + \frac{0.055}{0.945}\right)^{\frac{21.15}{5.6}} - 1\right] =$$

$$= 1{,}820\ (1.0582^{3.78} - 1) = 435.$$

If these rockets had to lift a man, they would have to be arranged in four rows (e.g. 100, 106, 112 and 117 units), whereas the acceleration exerted upon him would not be very great.*

It is now time to examine the physiological effect of accelerations different from $g = 9.81$ m/sec^2. The inner ear is the organ which is sensitive to velocity gradients. The ability of the human body to absorb this effect varies according to the circumstances; I will examine some of these.

Merry-go-round. The radius of the rotating ring is 4 m and the suspension of the seats is 2 m long. It accomplishes one complete rotation in 6.5 sec and the seats are deflected outward by 1.15 m; therefore the radius of the circular path is 5.15 m. The circumferential velocity is 5.5 m/sec and the centrifugal acceleration 5 m/sec^2. Taking into account
35 the vertical direction of gravity I will obtain the resultant acceleration $\sqrt{9.8^2 + 5^2} = 11$ m/sec^2. Although its vector creates an incline angle of 26.6° with the vertical, a passenger with shut eyes can point correctly in the vertical direction.

Banked flight along a curved trajectory. Contrary to the previous case the pilot experiences a different feeling. The earth surface does not seem immobile to him, but it tilts.

Elevator. The acceleration effect created by an elevator descending with a velocity of 1 m/sec and brought to a stop within 0.2 m (i.e. $(2.5 + g)$ m/sec^2 for $^2/_5$ sec) will be much more unpleasant than at a jump into water when a deceleration of $g = 25$ m/sec^2 acts during the same time interval ($^2/_5$ sec).

In general, a moderate roll (e.g., of a ship) is more unpleasant from the acceleration viewpoint than sudden braking. Moreover, the human reactions to these velocity-gradient phenomena depend on other factors, e.g.: whether or not they are expected; whether or not they are experienced voluntarily, and so on. Experiments performed in 1924 in the U.S.A. in the course of aerobatics proved that the maximum acceleration a pilot can

* Cf. "Die Rakete," p. 28. 1928.

withstand depends on how long it acts upon him. A pilot safely endures short accelerations which exceed even 7.8 times the normal gravitational acceleration ($g = 9.81\ m/sec^2$). However, accelerations higher than 4.5 g and inflicted upon him for a few (10 to 12) seconds affect an airman badly: the brain lacks oxygen (blood does not reach it) and he feels that he is going blind.

It should be noted that people with high blood pressure resist high accelerations better and regain their normal health sooner.

A rocket may experience different types of motion — rectilinear or curvilinear — rotations with reference to either of its inertia axes, accelerations and decelerations; human senses react differently to all of these. Especially peculiar is the effect of the Coriolis acceleration, which occurs during a combined type of motion (when a body simultaneously advances and rotates around a certain axis). All these acceleration effects on men and animals were studied by many scientists (Purkinje, 1826; Mach, 1875; Tsiolkovskii, 1895; Prandtl, 1926; Saint-Cyr experiments, 1927; Harco and others).

Acceleration need not affect human senses seriously. However, decrease of speed causes fright at the first moment, which can be overcome by
36 practice. One who will participate in rocket spaceflights will have to be trained beforehand in a rotating laboratory. Thus he will be immune to acceleration effects.

The above review of experiments shows that a human organism can hardly take accelerations exceeding 4 g unless special protective devices are developed. Assuming that 4 g corresponds to the effective climb acceleration $4g - g \cong 30\ m/sec^2$ it is obvious that the cosmic velocity $v = 9{,}000\ m/sec$ can be reached in 300 seconds. However, the acceleration effect will not be dangerous even at higher velocities, e.g., $v = 11{,}160\ m/sec$, because under normal conditions it is decreased by the gravitational attraction by $\int_0^{300} g\,dt = 2{,}400\ m/sec$ and by the aerodynamic resistance by another 200 m/sec.

In order to shorten the takeoff time, special devices (e.g., braking springs on hydraulic pistons) should be designed. Thus the acceleration effect will be reduced as much as possible. Tsiolkovskii suggested setting the acceleration endurance limit at 10 g, and staying under it by placing the man into a water container during takeoff (managing breathing by a special oxygen-supply device).

Experiments with permissible acceleration. Experiments to find out how acceleration affects the human body were carried out in Breslau on 10 July 1928. A merry-go-round built by the firm "Willi Vorlop jun" (Hannover) served the purpose. The distance between the center of gravity of the man and the axis of rotation equaled 3.2 m. He completed 24 revolutions in a minute (or one revolution in 2.5 sec).

Under these conditions the centrifugal accelerations equaled: $a = 4\pi^2 \cdot \frac{r}{t^2} = 39.48 \cdot \frac{r}{t^2}$, where r — radius = 3.2 m; t — period of one revolution = = 2.5 sec. Accounting for the acceleration (g) of gravity I derive the expression for the over-all acceleration $b = \sqrt{g^2 + a^2}$ which renders $b = 23\ m/sec^2$ for the given numerical values. That means that the man experienced an acceleration of 2.3 g.

No interference with normal functioning of the heart, lungs or the brain was noticed. Neither consciousness nor memory was affected. On the other hand, the pressure of the body on the outer wall was hardly bearable. Arms and legs became heavier although they remained well-controllable. Free muscles, e.g., those of the cheeks, stiffened, especially when the head was turned. It was difficult to hold the head straight when unsupported. The center of the merry-go-round seemed to be located higher than it really was, although not as high as would correspond to the direction of the resultant force (but only by 40°).

37 During the following experiment the speed was increased to one revolution in 1.7–1.8 sec or 10 revolutions in 17.5 sec. This corresponds to a centrifugal acceleration of 42 m/sec^2 (or 4.3 g). Heart, lungs and memory functioned normally. The arms and legs worked but appeared to get heavier. The weight of the clothes seemed to increase. Moreover, pressure was exerted by the body on the outer wall. The center of the merry-go-round seemed to be located even higher (by 20 cm).

According to observations of Gillert and Kaiser, who carried out similar experiments with a merry-go-round in Adlershof, Germany, a man withstood without injury an acceleration of $4\frac{1}{2}$ g while facing the rotation axis, but looking outwards he lost consciousness in quite a short time.

b) Absence of accelerations

Everything inside a spacecraft moving only under the effect of inertia and gravity forces will become weightless. All bodies not attached to the frame of the craft will start floating in its inner space. Released outside the craft, they will move alongside with zero relative velocity. Liquids will become spheroidal and cease to exert pressure on the sides of their containers, and so on.

It is possible to create an artificial gravity by means of rotating the rocket or the compartment in it around a certain axis. The resulting centrifugal force will produce the desired acceleration effect. For example, for a rocket 100 m long and rotating at 10 m/sec, the acceleration will be $\frac{10^2}{50} = 2$ m/sec^2, i.e., $\frac{1}{5}$ g.

8. APPROXIMATE ROCKET THRUST CALCULATIONS. TSIOLKOVSKII'S EQUATIONS AND STATEMENTS

Having discussed the physical parameters incorporated in the basic equation of rocket motion (1), I will now examine different cases of rocket flight. Here are some of the approximate methods of calculating the flight of a craft propelled by reaction forces.

a) Vertical flight in a vacuum and outside any gravitational field

α) Takeoff. I cite equation (1) taking $R=O$, $g=O$ and supposing that the shell (empty fuel tank) is not jettisoned,

$$cdm = (M - m)\,dv. \tag{13}$$

Separating the variables, I will obtain

$$\frac{dv}{c} = \frac{dm}{M - m}.$$

38 Integrating this equation will give

$$\int \frac{dv}{c} = \int \frac{dm}{M - m} + C$$

or

$$\frac{v}{c} = -\ln(M - m) + C.$$

Here ln is the natural logarithm.
At the beginning of the flight $m = 0$ and $v = 0$. Therefore

$$C = \ln M.$$

Then

$$\frac{v}{c} = \ln M - \ln(M - m) = \ln\frac{M}{M - m}$$

or

$$v = c\ln\frac{M}{M - m}. \tag{14}$$

The velocity v will attain its maximum value v_{1max} when all the propellant is consumed, i. e., when $m = m_1$, m_1 being the total propellant mass.

Denoting the remaining rocket mass as M_1 yields the following form of equation (14):

$$v_{1max} = c\ln\frac{M_1 + m_1}{M_1}$$

and, defining $q_1 = \frac{m_1}{M_1}$, I obtain finally

$$v_{1max} = c\ln(1 + q_1) \tag{15}$$

which renders

$$q_1 = \exp(v_{1max}/c) - 1 \qquad (16)$$

Since equation (15) was first derived by Tsiolkovskii, I refer to it as "Tsiolkovskii's first equation." Its meaning is as follows:

Statement 1. Rocket flight velocity in a vacuum, with zero gravitation, proportional to the jet velocity (c) and increases with the increase in the ratio of the propellant mass to the total mass of the rocket. From equation (16) follows:

Statement 2. The higher the jet velocity, the less propellant is needed in a rocket to attain a certain flight velocity.

39 The lower curve on Figure 16 represents equation (15). The q_1-ratios are plotted on the axis of abscissas, v_{1max} on the axis of ordinates. The curve shows that just after the launch the velocity increases rapidly and then this increase slows down.

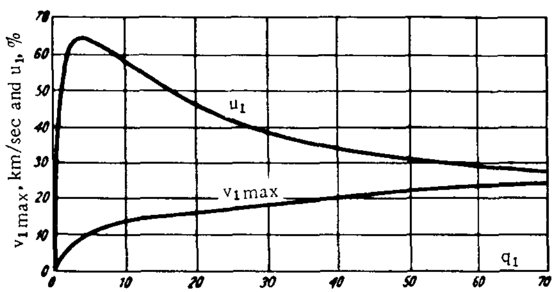


FIGURE 16. Flight velocity and utilization of the rocket.

I define — after Tsiolkovskii — utilization (u_1, %) as the ratio of the work of the rocket to the work of the propellant.

The work of the rocket equals:

$$T_1 = \frac{v^2_{1max}}{2} M_1; \text{ or, taking (15) into account:}$$

$$T_1 = \frac{c^2}{2} M_1 \, [\ln(1+q_1)]^2.$$

The work of the propellant is:

$$T_1' = \frac{c^2}{2} m_1.$$

Thus the utilization is:

$$u_1 = \frac{T_1}{T_1'} = \frac{M_1}{m_1} [\ln(1+q_1)]^2 = \frac{1}{q_1} [\ln(1+q)]^2 \qquad (17)$$

This is Tsiolkovskii's second principal equation. It is represented by the upper curve in Figure 16. Ratios q_1 are plotted on the axis of abscissas and the utilization u_1 on the axis of ordinates.

On the basis of equation (17) the following statement can be made:

Statement 3. The rocket utilization increases with the ratio q (fuel mass to rocket mass), reaching a maximum (64.75%) at $q_0 = 4$, and then decreasing.

Results computed from formulae (15) and (17) for jet velocities of 5,000 and 4,000 m/sec are presented in Table 8.

40 TABLE 8. Ratios (q) of propellant mass to rocket mass for different cases of flight

Rocket velocity v_p, m/sec		Gravitation-free medium			Medium with gravitation			
$v_s = 5{,}000$ m/sec	$v_s = 4{,}000$ m/sec	takeoff		takeoff and descent	takeoff q_3		takeoff and descent q_4	
		q_1	k_p	q_2	b = 4g	b = 10g	b = 4g	b = 10g
0	0	0	0	0	0	0	0	0
472.5	378	0.1	8.87	0.21	0.13	0.11	0.29	0.24
910	728	0.2	16.55	0.44	0.27	0.22	0.62	0.50
1,310	1,048	0.3	22.9	0.69	0.42	0.34	1.01	0.79
1,680	1,344	0.4	28.2	0.96	0.56	0.45	1.45	1.11
2,025	1,620	0.5	32.8	1.25	0.72	0.57	1.94	1.46
2,345	1,876	0.6	36.7	1.56	0.87	0.68	2.49	1.84
2,645	2,116	0.7	40.0	1.89	1.03	0.80	3.10	2.25
2,930	2,344	0.8	42.9	2.24	1.19	0.92	3.78	2.69
3,210	2,568	0.9	45.8	2.61	1.35	1.04	4.51	3.16
3,465	2,772	1.0	48.0	3.00	1.52	1.16	5.32	3.66
4,575	3,660	1.5	55.8	5.25	2.38	1.77	10.44	6.65
5,490	4,392	2	60.3	8.00	3.31	2.39	17.55	10.5
6,900	5,520	3	63.5	15.0	5.32	3.66	38.95	20.7
8,045	6,436	4	64.75	24.0	7.50	4.97	71.32	34.6
8,960	7,168	5	64.1	35	9.84	6.31	116.5	52.4
9,730	7,784	6	63.0	48	12.30	7.67	176	74.2
10,395	8,316	7	61.7	56	14.89	9.16	251	100
10,985	8,788	8	60.5	80	17.58	10.46	344	130
11,515	9,212	9	58.9	99	20.38	11.92	463	165
11,990	9,592	10	57.6	120	23.27	13.32	588	204
13,865	11,092	15	51.2	255	38.95	20.71	1,595	470
15,220	12,176	20	46.3	440	56.35	28.35	3,289	861
17,170	13,736	30	39.3	960	95.27	44.23	9,268	2,045
22,400	17,920	50	31.0	2,700	185.7	77.60	34,844	6,176
26,280	21,042	100	21.0	10,200	462.2	168.8	214,529	28,156
30,038	24,032	193	14.4	37,635	1,102	345.3	1,217,690	119,929
∞	∞	∞	0	∞	∞	∞	∞	∞

Note. The forthcoming calculation shows that the maximum u_1 is obtained at $q_1=3.92155$ and equals 64.76%.

I now find the derivative of (17):

$$u_1=\frac{[\ln(1+q_1)]^2}{q_1}$$

$$u_1'=\frac{\frac{2q_1\ln(1+q_1)}{1+q_1}-[\ln(1+q_1)]^2}{q_1^2}=\frac{\ln(1+q_1)}{(1+q_1)q_1^2}\,[2q_1-(q_1+1)\ln(1+q_1)].$$

The condition for maximum is:

$$2q_1-(1+q_1)\ln(1+q_1)=0$$

or converted to common logarithms:

$$f(q_1)=2q_1-(1+q_1)\,\lg\frac{(1+q_1)}{M}=0.$$

41 I carry out the calculation using Newton's method and first compute the derivative of the previous $f(q_1)$:

$$f'(q_1)=1-\frac{\lg(1+q_1)}{M}$$

1st approximation for the root $q_1=4$:

$$\frac{f(q_1)}{f'(q_1)}=0.0776;$$

2nd approximation for the root $q_1=4-0.0776=3.9224$:

$$\frac{f(q_1)}{f'(q_1)}=0.00085;$$

3rd approximation for the root $q_1=3.9224-0.00085=3.9215$:

$$\frac{f(q_1)}{f'(q_1)}=0.000007;$$

4th approximation for the root $q_1=3.92155+0.000007=3.921557$.

The maximum $u_1=0.6476$ (or 64.76%) is found at $q_1=3.92155$.

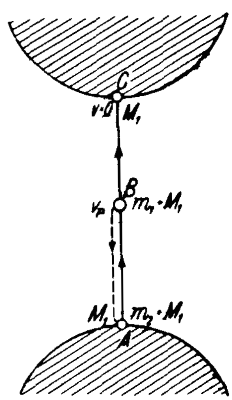


FIGURE 17

If a rocket moves with constant acceleration b, the time it takes to consume all its propellant is:

$$t=\frac{v_{1\max}}{b}. \qquad (18)$$

I now determine the flight velocity which corresponds to the best utilization. The ratio between the work produced to the work consumed can be written in the form:

$$u_1=\frac{\frac{m_1c^2}{2}-\frac{m_1(c-v)^2}{2}}{\frac{m_1c^2}{2}}=\left(2-\frac{v}{c}\right)\frac{v}{c}.$$

The highest value of u_1 will be achieved when $2-\frac{v}{c}=\frac{v}{c}$ or at $v=c$.*

β) Ascent and descent assisted by reaction thrust. I now examine the following case (Figure 17). A rocket is launched from point A in a medium where no atmosphere or gravity exist. Consuming a part of its fuel it reaches point B at a certain velocity $v_{2\,max}$. It then turns 180° around its transverse axis and by means of retro-thrust
42 decelerates down to zero velocity at point C (BC = AB). This case is essentially equivalent to the following one. A rocket launched from A with an initial velocity $v_{1\,max}$ arrives at B with zero velocity and, falling back, regains the same v_{1max} at A (the return trip is represented by the dashed line).

I denote the propellant reserves at the takeoff by m_2.

The propellant mass necessary for a rocket of mass M_1 to take off from A and arrive at B is

$$m_1 = q_1 \cdot M_1;$$

the same for the propellant mass m_1 from A to B is

$$m_1' = q_1 \cdot m_1 .$$

To decelerate the mass M_1 from B down to A, we need

$$m_1 = q_1 \cdot M_1 .$$

The total propellant mass is

$$m_1 = 2m_1 + m_1' = 2q_1M_1 + q_1m_1 = q_1\,(2M_1 + m_1).$$

Dividing both sides of the equation by M_1 and denoting $\frac{m_2}{M_1} = q_2$, I have:

$$q_2 = q_1\ (2 + q_1).$$

But

$$2q_1 + q_1^2 = (1 + q_1)^2 - 1 .$$

Thus, finally

$$q_2 = (1 + q_1)^2 - 1. \qquad (19)$$

This leads to the following statement.

Statement 4. For a rocket ascending and returning by means of reaction of gases in a medium without gravity and atmosphere, the ratio of the propellant mass to the mass of the rocket equals

$$(1 + q_1)^2 - 1,$$

where q_1 is that same ratio, necessary only for ascent.

* Professor B. Stechkin also worked on the problem of determining thrust and efficiency of a jet engine working in air ("Teoriya vozdushnogo reaktivnogo dvigatelya" (Theory of Jet Engine). – "Tekhnika vozdushnogo flota," No. 2. 1929, e.g., at $v = 600$ m/sec, $u_1 = 0.182$).

Substitution of the corresponding ratios q_1 in the formula (19) yields the q_2-values (see Table 8). Comparison of q_1 and q_2 columns shows the remarkable increase of the ratio of propellant-to-rocket mass in the case of a descent assisted by a decelerating retro-thrust.

b) Vertical flight in a gravitational field

α) Ascent. Assuming first that the gravitational acceleration equals g and is constant, I begin to examine the case of a rocket taking off in a gravitational field.

The burning time of a given propellant mass does not depend on whether
43 or not the rocket travels through a gravitational field. Thus, analogously to equation (18):

$$t = \frac{v_{1max}}{b} = \frac{v_{3max}}{b-g} \qquad (20)$$

where v_{3max} is rocket velocity at the moment when all the propellant is consumed. Obviously $v_{3max} < v_{1max}$.

Taking equation (15) into account I obtain from (20):

$$v_{3max} = c\left(1 - \frac{g}{b}\right) \ln (1 + q_1). \qquad (21)$$

Should I wish to obtain the same velocity as in the non-gravitational case, i. e., v_{1max}, I would have to take a certain ratio q_3 of the propellant mass m_3 to the mass M_1 of the rocket. This ratio is determined by the formula:

$$v_{1max} = c\left(1 - \frac{g}{b}\right) \ln (1 + q_3) \qquad (22)$$

which yields

$$q_3 = e^{\frac{v_{1max}}{c} \frac{b}{b-g}} - 1; \qquad (23)$$

but, according to (16):

$$e^{\frac{v_{1max}}{c}} = q_1 + 1.$$

Therefore (23) is converted into:

$$q_3 = (q_1 + 1)^{\frac{b}{b-g}} - 1. \qquad (24)$$

This is Tsiolkovskii's third equation. Together with equation (21) it leads to the next statement.

S t a t e m e n t 5. The smaller the gravitational force, the higher will be the flight velocity. Besides, to reduce q_3 it pays to increase b, i.e., to impart to the rocket a higher acceleration in order to facilitate a faster escape from the terrestrial gravitational field.

Values of q_3 computed from (24) are presented in Table 8.

I now calculate two examples to find q_3 and the ascent time, one for $b=10g$, the other for $b=4g$ (see sec. 7). Taking the rocket velocity as

$$v_{1\,max}=11{,}990 \text{ m/sec}; \quad c=5{,}000 \text{ m/sec}.$$

44 F i r s t e x a m p l e:

$$b=10g.$$

From relation (22):

$$11{,}990=5{,}000\left(1-\frac{1}{10}\right)\ln(1+q_3),$$

hence

$$q_3=13.32.$$

Ascent time

$$t=\frac{11{,}990}{10g-g}=\sim 133 \text{ sec}.$$

S e c o n d e x a m p l e:

$$b=4g;$$

$$11{,}990=5{,}000\left(1-\frac{1}{4}\right)\ln(1+q_3);$$

$$q_3=23.27; \quad t=400 \text{ sec}.$$

Without gravity the same velocity could be attained at $q_1=10$ during the following ascent time: in the first case

$$t=\frac{11{,}990}{10g}=\sim 120 \text{ sec}$$

and in the second case

$$t=\frac{11{,}990}{4g}=\sim 300 \text{ sec}.$$

I will now compare the work done by the rocket during its travel in a gravitational field to that done in a gravitation-free medium.

I assume, the same as Tsiolkovskii, that gravitational acceleration (g) does not change with altitude; then we obtain results much less favorable than in reality.

From mechanics, the work done by a rocket in a gravitational field is:

$$T_3 = \frac{v_{3max}^2}{2} M_1,$$

and in a medium without gravitation

$$T_1 = \frac{v_{1max}^2}{2} M_1.$$

Taking (20) into account, the ratio of these two quantities of work is

$$\frac{T_3}{T_1} = \frac{v_{3max}^2}{v_{1max}^2} = \left(\frac{b-g}{g}\right)^2 = \left(1-\frac{g}{b}\right)^2 \qquad (25)$$

This formula confirms statement 5.

β) Ascent and descent by reaction of gases. To
45 determine the ratio q_4 of the propellant mass to the rocket mass in this case, I make use of the formula (19), in which q_3 is substituted for q_1. From the relation (24)

$$q_4 = \left\{1 + \left[(q_1+1)^{\frac{b}{b-g}} - 1\right]\right\}^2 - 1 = (q_1+1)^{\frac{2b}{b-g}} - 1. \qquad (26)$$

I have calculated the q_4-values for different q_1 and for $b = 10g$ and $b = 4g$. They appear in Table 8.

Values of q_1, q_2, q_3, q_4 for different c (5,000 and 4,000 m/sec) and b (10g and 4g) are plotted versus v in Figure 18.

c) Inclined ascent in a gravitational field

If a rocket possessing its own acceleration (b) is launched vertically (i.e., along a radius of the planet), its effective acceleration in a medium subjected to the influence of a constant gravitational acceleration (g) will be $b - g$.

But if it follows an inclined trajectory, its relative acceleration will be higher. For example (Figure 19), if the angle the rocket path makes with the vertical is γ, its relative acceleration will be

$$b_1 = \sqrt{b^2 + g^2 - 2bg \cos \gamma}. \qquad (27)$$

In the limit, its acceleration in horizontal flight will be:

$$b_2 = \sqrt{b^2 - g^2},$$

and

$$\cos \gamma = \frac{g}{b}.$$

46 I will now examine how the work of a rocket changes at different ascent angles (keeping the initial fuel reserves and burning time constant).

One must keep in mind that the work is proportional to the square of the acceleration. Thus the ratio of the work done in an inclined ascent to that in vertical ascent is

$$u' = \frac{b^2 + g^2 - 2bg \cos \gamma}{(b - g)^2}. \qquad (28)$$

I denote

$$b = ng.$$

Then

$$u' = \frac{n^2g^2 + g^2 - 2ng^2 \cos \gamma}{g^2 (n-1)^2} = \frac{(n^2 + 1 - 2n \cos \gamma)}{(n-1)^2}$$

In the limit case (that of horizontal flight):

$$\cos \gamma = \frac{g}{b} = \frac{1}{n},$$

then

$$u' = \frac{n+1}{n-1}.$$

Table 9 presents values of u' for different γ, for

$$\cos \gamma = \text{from } 1 \text{ to } \frac{1}{n},$$

and

$$n = 4 \text{ and } n = 10.$$

The data of Table 9 are plotted in Figure 20 in the form of two curves: one for u' at $n = 4$, and the other at $n = 10$.

Both these graphs and the relation (28) lead to the following conclusion:

Statement 6. *For flight in a vacuum it is advantageous to keep the launching angle and the acceleration as low as possible.*

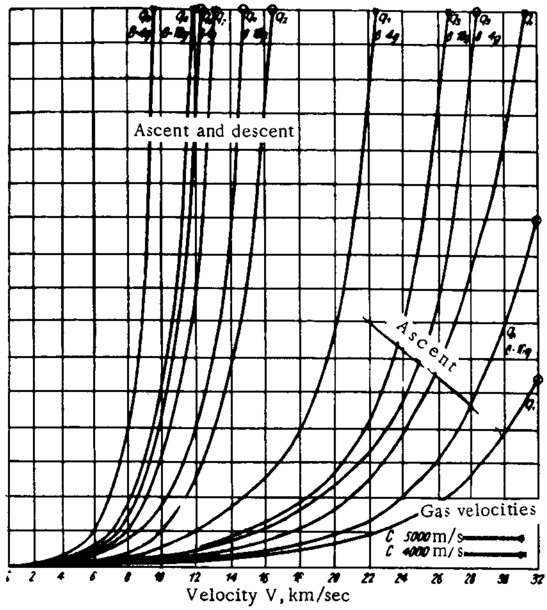


FIGURE 18. Rocket weights for different cases of rocket flight

(46)

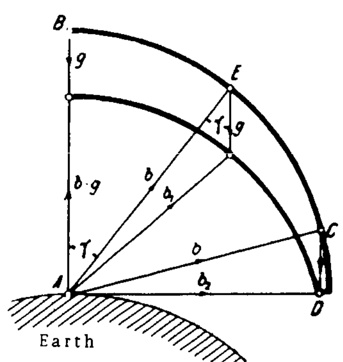


FIGURE 19. Inclined ascent of a rocket

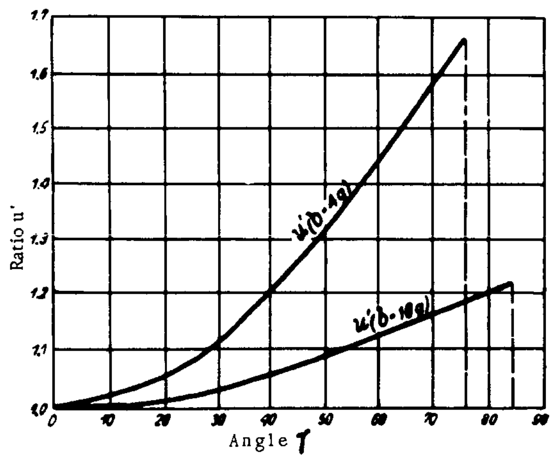


FIGURE 20. Work of a rocket in inclined ascent

TABLE 9

γ	u'	
	n = 4	n = 10
0°	1.00	1.00
10°	1.02	1.00
20°	1.05	1.01
30°	1.12	1.03
40°	1.20	1.06
50°	1.32	1.09
60°	1.44	1.12
70°	1.59	1.16
75°30	1.66	1.19
80°	–	1.20
84° 50'	–	1.22

47 **d) Launch in a medium with gravity and atmosphere**

Atmospheric resistance reduces the rocket velocity. The smaller the launching angle, the more energy the rocket will have to spend to overcome this aerodynamic resistance. Thus the air-drag factor cancels the advantage of the inclined launch, which otherwise could diminish the effect of gravity.

I will now evaluate the influence of the atmosphere in a vertical launching.

The upward acceleration without taking the atmosphere into account is

$$\frac{dv}{dt} = b - g_0 \frac{r_0^2}{r^2},$$

where r_0 — the Earth's radius, and $r = r_0 + h$, where h — the ascent altitude of the rocket.

But $\frac{dr}{dt} = v$ and therefore

$$vdv = \left(b - g_0 \frac{r_0^2}{r^2}\right) dr.$$

Integrating, I obtain

$$\frac{v^2}{2} = br + \frac{g_0 r_0^2}{r} + C .$$

The constant C is determined from the condition that at the Earth's surface $r = r_0$ and $v = 0$, thus

$$C = - r_0 (b + g_0)$$

and

$$\frac{v^2}{2} = (r - r_0)\left(b - g_0 \frac{r_0}{r}\right). \qquad (29)$$

The overall atmospheric resistance can be represented by the resistance of an equivalent air layer 50 km thick with an average specific weight of $0.2 = \delta$ throughout it.

The resistance is given by (see (12)):

$$R = \frac{\delta \cdot v^2}{g} \cdot F \cdot i = \frac{0.2}{10} \cdot v^2 F \cdot i.$$

The deceleration caused by this resistance is

$$b_a = \frac{R}{M} = \frac{0.2}{10} \cdot v^2 \cdot \frac{F}{M} \cdot i. \tag{30}$$

Adopting Hohmann's assumptions I will take a 600-kg load per m^2 of the rocket frontal area; its shape factor $i = 0.12$ and the average ascent speed through this layer is about 0.775 km/sec.

Then

$$b_a = \frac{0.2}{10} \cdot \frac{1}{600} \cdot 0.12 \cdot 775^2 = 2.4 \text{ m/sec}^2.$$

48 Therefore, instead of acceleration b the rocket will receive an acceleration of

$$(b - 2.4) \text{ m/sec}^2.$$

For example, $b = 40$ will yield:

$$b - 2.4 = 37.6 \text{ m/sec}^2.$$

Computing from (29), for $r = 6{,}370$ we obtain

$$\frac{v^2}{r} = 50\left(0.0376 - 0.0098 \cdot \frac{6{,}370}{6{,}420}\right) = 1.395 \text{ km}^2/\text{sec}^2$$

instead of

$$50\left(0.04 - 0.0098 \frac{6{,}370}{6{,}420}\right) = 1.515 \text{ km}^2/\text{sec}^2,$$

or

$$v = \sqrt{2 \cdot 1.395} = 1.67 \text{ km/sec}$$

instead of

$$v \cdot \sqrt{2 \cdot 1.515} = 1.74 \text{ km/sec}.$$

Thus the flight duration will be

$$t = \frac{v}{b - b_a - g} = \frac{1{,}670}{37.6 - 9.8} = 60 \text{ sec}$$

instead of

$$\frac{1{,}740}{40-9.8}=57.6 \text{ sec.}$$

The mass ratio will be (from (24))

$$q_4=(q_1+1)^{\frac{b}{b-g-2.4}}-1 \qquad (31)$$

instead of

$$q_3=(q_1+1)^{\frac{b}{b-g}}-1.$$

Assuming, for example, $q_1 = 10$, $b = 4g$, I will get:

$$q_4 = 10.8$$

instead of

$$q_3 = 10.2.$$

Hohmann has compiled into table form a comparison of mass ratios at different flight accelerations and jet velocities (c).

49 TABLE 10

	q_3	q_4	q_3	q_4	q_3	q_4
Accelerations, m/sec²	30	30	100	100	200	200
Flight time, sec	448	456	117	123	57	64
c = 4,000 m/sec	28.7	30	18.7	22	17.2	25
c = 5,000 m/sec	14.6	15	10.4	12	9.8	13
Final velocity v, km/sec	9.68	–	10.65	–	10.89	–

Examination of equation (30) reveals various means of reducing the aerodynamic drag effect: a) increasing the mass of the rocket; b) diminishing its cross-sectional area; c) lowering flight acceleration; d) improving the aerodynamics of the rocket (decrease i). In general, even at b = 10g the masses increase (see Table 8) by 18–15%, whereas at b=4g only by 5–3%.

9. THE LATEST STUDIES IN ROCKET FLIGHT

The classic work by Tsiolkovskii was followed by many works by scientists who studied in detail many specific problems in reactive propulsion and spacecraft design. I will mention some of them: 1) Esnault-Pelterie

"Considérations sur les résultats d'un allègement indéfini des moteurs," published in France in 1913; 2) Robert Goddard "A Method of Reaching Extreme Altitudes," which appeared in the U. S. A. in 1919; 3) Hermann Oberth "Die Rakete zu den Planetenräumen," a book published in Germany in 1924; 4) Hohmann "Die Erreichbarkeit der Himmelskörper," which appeared in Germany in 1925.

Further, in the years 1926–1928 works of Valier, Lorenz, Lademann, Manigold, Scherschevsky, Pirquet, Crocco and new publications by Oberth, Esnault-Pelterie and many others appeared.

I shall just point out the principal contributions of certain authors, drawing the attention of the reader interested in details to the bibliography at the end.*

50 a) Esnault-Pelterie's work

Esnault-Pelterie writes the basic equation of rocket motion (1) in the form:

$$cdm = -Mdv.$$

Further, he accounts for the gravitational work and develops the calculation of a rocket traveling from the Earth to the Moon and back to Earth. He distinguishes between three distinct phases in such a flight:

1. The rocket takes off from the Earth's surface and accelerates up to the velocity which enables it to escape from the terrestrial gravitational field.
2. Combustion (energy release) is cut off and the rocket is driven forward by inertia.
3. At a certain point in the approach to the Moon the rocket turns through 180° around its transverse axis and the reignited engine decelerates the craft down to zero velocity at the Moon surface. An analogous sequence of events takes place during the return trip. The author assumes a very low permissible acceleration, namely 1.1g.

The initial rocket weight is taken as 1,000kg, of which the propellant has a share of 300kg. Under such conditions the necessary jet velocity has to be 65,300 m/sec and the propellant must produce $512 \cdot 10^3$ kcal per kilogram, which no existing fuel can supply. Therefore, Esnault-Pelterie suggests using radium for fuel, which can supply this energy and more.

b) Goddard's work

Goddard introduces an original method of solving the basic equation of rocket motion (1):

$$c\,(1-k)\,dm = (M-m)\,dv + [R + g\,(M-m)]\,dt. \qquad (1)$$

* A similar account of the history, technological theory and literature pertaining to interplanetary communications can be found in N.Rynin's "Mezhplanetnye soobshcheniya" (Interplanetary Flight and Communication), No.1 "Mechty, legendy i pervye fantazii" (Dreams, Legends, and Early Fantasies); No.2 "Kosmicheskie korabli v fantaziyakh romanistov" (Spacecraft in Science Fiction); No.3 "Luchistaya Energiya v fantaziyakh romanistov" (Radiant Energy in Science Fiction); No.4 "Rakety" (Rockets). [Translated by IPST, TT-50111, TT-50112, TT-50113, TT-50114.]

He assumes that a rocket flight can be realized if the initial rocket mass M does not exceed a reasonable maximum value. This leads to a definition of a constraint imposed on the problem: the mass (m) ejected at the time (t) should not be lower than a certain minimum value. This condition of minimum man is used while integrating the equation of motion (1).

The following reasoning proves that such a minimum of mass does exist (granted that a given mass has a predetermined velocity at a given altitude). If at a certain intermediate altitude the ascent speed is excessively high, the air resistance (which depends on the square of the velocity) will be high as well. On the other hand, if the ascent speeds are too low, the appropriate jet reaction will have to overcome the Earth's gravity for too
51 long a time. Both cases demand excessive quantities of propellant. It is obvious now that the ascent speed has a suitable value for each altitude. To put it in a different way: the unknown function

$$v = f(h),$$

where h is the altitude, has to be determined in order to keep m at the minimum value.

Integration of the equation (1) leads to a variational problem which seems insoluble to Goddard.* Instead he suggests the following approximate solution:

He divides the altitude h into a large number (n_0) of intervals and assumes that R, g and the upward acceleration are constant throughout each interval. Let v in each interval be given by $v = bt$, where b — constant acceleration. Then equation (1) assumes the form

$$\frac{dm}{dt} = \frac{(M-m)(b+g)+R}{c(1-k)},$$

and its solution is

$$m = e^{-\frac{b+g}{c(1-k)}t} \cdot \frac{M(b+g)+R}{b+g}\left[e^{\frac{b+g}{c(1-k)}} + C\right].$$

The constant C is found to be -1 from the condition that at $t=0$ we have $m = 0$. Thus

$$m = \left(M + \frac{R}{b+g}\right)\left(1 - e^{-\frac{b+g}{c(1-k)}t}\right). \qquad (32)$$

This equation is applicable to every altitude interval throughout which R, g and b remain constant.

Introducing the condition that the payload that reaches the highest point of the trajectory equals 1 lb, the equation can be further simplified.

In this case the initial rocket mass necessary to lift this unit payload up to the desired altitude will be the sum of n_0 initial masses for the separate intervals.

* G. Hamel in Berlin solved this problem later and found a minimum v of about 1,000 to 1,100 m/sec at an altitude of h = 100 km.

Assuming the final mass in each interval to be $M = m = 1$ and substituting this value in equation (32) Goddard obtains the mass at the beginning of the interval under examination

$$M = \frac{R}{b+g}\left(e^{\frac{b+g}{c(1-k)}t} - 1\right) + e^{\frac{b+g}{c(1-k)}t} \qquad (33)$$

52 If R and g equal zero, then

$$M = e^{\frac{bt}{c(1-k)}}. \qquad (34)$$

The ratio of masses in equations (33) and (34) indicates the effect of R and g on the mass increase. When this ratio is minimum, M will also be minimum for the same interval. The total initial mass necessary to lift 1 lb of payload up to a given altitude will be the product of the minimum masses (M), obtained for each interval.

Goddard takes the following values for his calculations: $k = \frac{1}{15}$; $c(1-k) = 2{,}134$ m/sec (or even less). He derives the aerodynamic resistance from relation (11). He divides the atmosphere (which is assumed to extend up to an altitude of 2,685 km) into nine intervals and chooses accelerations (b) as $b = 15$ m/sec^2 and 45.7 m/sec^2.

The results of Goddard's calculations of the initial-to-final mass ratio for a powder rocket are presented in Table 11. He carries out the computations for $c(1-k) = 2{,}134$ m/sec and $b = 45.7$ m/sec^2. Knowing b, he is not obliged to compute the minima of the masses but determines them directly from equations (33) and (34), in which all the right-side parameters are known for each interval. The time t of every interval is found from mechanics — according to the condition of motion of bodies projected upward in a vacuum. The equation is $h = v_0 t + \frac{1}{2} b_1 t^2$, where v_0, h and b_1 are known.

TABLE 11

Altitude, km	56	186	704	∞
Flight time, sec	114.13	265.93	475.23	∞
$\frac{M}{M_1}$	3.665	6.40	12.33	602.0

c) Oberth's work

Oberth writes the basic equation of rocket motion (1):

$$c(1-k)\,dm = (M-m)\,dv + [R + g\,(M-m)\,dt] \qquad (1)$$

in the form:

$$cdm + mdv + Qdt = 0;$$

let

$$k = 0; \quad M - m = m'; \quad Q = R + g\,(M - m).$$

From now on I will explain Oberth's reasoning, using the notations of
53 equation (1) and assuming $k = 0$ in it. The term cdm is transferred to the right because the ejection velocity is opposite in sign to the flight velocity:

$$(M - m)\,dv + [R + g\,(M - m)]\,dt + cd\,(M - m) = 0$$

or

$$m'dv + Qdt + cdm' = 0. \tag{35}$$

At first Oberth treats the rocket motion inside the terrestrial atmosphere and, as Goddard did, determines the optimum velocity, i. e., the velocity at which: 1) momentum $(M - m)dv$ is constant, and 2) propellant consumption dm is minimum.

Let a rocket at an altitude h pass through an air layer of such a small thickness dh that through this interval the air density and the rocket mass m' remain practically constant, whereas the momentum shows a constant increase by mdv.

The time of traversing this layer is $dt = \frac{dh}{v}$ and the equation (35) changes into:

$$\frac{m'dv}{dh} + \frac{Q}{v} + c\,\frac{dm'}{dh} = 0. \tag{36}$$

Assuming m'dv and dh constant I differentiate with respect to v:

$$\frac{d\left(\frac{Q}{v}\right)}{dv} + \frac{dc}{dv} \cdot \frac{dm'}{dh} + c\,\frac{d\left(\frac{dm'}{dh}\right)}{dv} = 0.$$

The supposition $c = \text{const}$ obliges the second term of this equation to equal zero.

The condition of minimum propellant consumption gives:

$$\frac{d\left(\frac{dm'}{dh}\right)}{dv} - \frac{1}{dh}\,d\left(\frac{dm'}{dv}\right) = 0$$

and then

$$\frac{d\left(\frac{Q}{v}\right)}{dv} = 0.$$

But $Q = R + gm'$, gm' being the body weight and R its aerodynamic resistance:

$$R = k \cdot F \cdot \beta \cdot v^2.$$

Here k is the aerodynamic drag coefficient, which depends on the shape of the rocket and its velocity v.

54 Substituting these values in the expression for Q, we obtain

$$\frac{Q}{v} = F\beta k v + \frac{m'g}{v},$$

and in the expression of the derivative we have

$$\frac{d\left(\frac{Q}{v}\right)}{dv} = -\frac{m'g}{v^2} + F\beta\left(k + v\frac{dk}{dv}\right).$$

Setting this expression equal to zero I obtain the optimum velocity $(\bar{v})$

$$\bar{v}^2 = \frac{m'g}{F\beta\left(v\frac{dk}{dv} + k\right)}. \qquad (37)$$

Multiplying all the terms of the equation (36) by $\frac{dh}{m'c}$ and substituting the optimum velocity $\bar{v}$ instead of v results in Oberth's basic equation. This equation describes the rocket motion for minimum propellant consumption:

$$\frac{d\bar{v}}{c} + \frac{Qdt}{m'c} + \frac{dm'}{m'} = 0. \qquad (38)$$

The following quantities enter this equation in connection with (37):

$$g = 9.81 \cdot \frac{r^2}{(r+h)^2},$$

$$\beta = \beta_0 e^{-\frac{h-h_0}{H'}},$$

$$k = \text{const} \quad \text{at } v > 460 \text{ m/sec},$$

$$H' = 6{,}300 \text{ m}.$$

Computations based on the method revealed above bring Oberth to the conclusion that an acceleration of 30 m/sec^2 has to be imparted to a manned spacecraft in 332 sec in order to pierce the double envelope of the Earth's gravity and aerodynamic resistance. Thus it will reach an altitude of 1,653 km and a velocity of 9,954 m/sec. The escape into space will be guaranteed at this point because the computed velocity exceeds the parabolic one.

To overcome the Earth's gravity, the rocket experiences at the beginning of these 332 sec a deceleration of 9.81, and at the end of this interval 6.17 m/sec^2 (on average 8 m/sec^2), thus losing 2,656 m/sec of velocity. The atmospheric drag losses amount to 200 m/sec. Therefore the rocket should develop a velocity of 9,954 + 2,656 + 200 = 12,810 m/sec. If the rocket
55 follows a curvilinear and not a vertical path, this velocity can be attained in $^5/_6$ of the previous time interval, namely in 260 sec. The gravity deeeleration effect will result in 2,000 m/sec (instead of 2,656), and the thrust will speed the rocket up to 12,100 m/sec.

An unmanned craft can undergo higher accelerations and, making its way up in a shorter time, it will "spend" only about 800 m/sec on the decelerating effect. Should this effect be absent, the rocket would attain the parabolic velocity of 10,923 m/sec at the altitude of 280 km, where the gravitational acceleration equals 8.996 m/sec^2. Therefore, the overall velocity needed will be 10,923 + 800 = 11,723 m/sec.

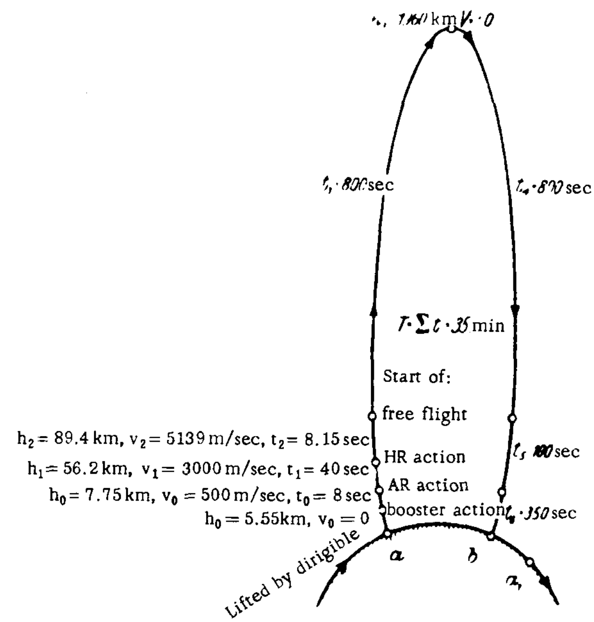


FIGURE 21. Rocket flight, after Oberth

Oberth suggests a three-stage rocket design (Figure 5) for an unmanned investigation of the upper atmosphere. The launch is to be carried out in the following sequence (Figure 21).

The rocket is lifted by means of dirigibles from a given point (a) on the Earth's (or sea) surface up to an altitude of h'_0 = 5.55 km. There it is released from the dirigibles and a booster rocket takes it in 8 sec up to an altitude h_0 = 7.75 km; a velocity v_0 = 500 m/sec is imparted to it at the same time. At that altitude the booster is jettisoned and the second-stage alcohol rocket (AR) takes over. In 40 seconds it takes the craft up to an altitude of $h_1 = 56.2$ km, speeding it up to a velocity $v_1 = 3{,}000$ m/sec. Now it is the turn of the hydrogen engine (after the AR has been jettisoned). It brings the vehicle in 8.15 sec up to the altitude $h_2 = 89.4$ km, imparting to it a velocity $v_2 = 5{,}139$ m/sec. Here the last jettisoning takes place and the upper cabin, equipped with stabilizers, performs a free ascent up to the highest point of the trajectory at $h_3 = 1{,}960$ km. Then tracing an ellipse it descends by parachute to a point (b) on the Earth's surface. This point will be behind (to the west of) the launch point (a), which has passed in the meantime to point (a_1).

Weights of the rocket components are the following:

HR — Hydrogen rocket:		
Head including instrumentation and parachutes	3.6 kg	
Propellant	3.3 kg	
Full weight		6.9 kg
56 AR — Alcohol rocket:		
Shell without propellant	51.2 kg	
Propellant	487.77 kg	
Full weight		538.97 kg
Weight of both rockets		545.87 kg
Booster with propellant		220 kg
	Total	765.87 kg

Concluding his work Oberth introduces a few ideas about construction of a manned spacecraft, which will circumnavigate the Moon and return to Earth (performing a parachute descent).

The weight of this suggested craft:

for one astronaut	300 t
for two astronauts	400 t

d) Hohmann's work

Hohmann carries out his calculations for the following gas-jet velocities: 2,000, 2,500, 3,000, 4,000 and 5,000 m/sec. He assumes a solid propellant of conical shape (Figure 3). The cabin is situated at the top of the cone, which is to burn gradually upwards. However, only low gas velocities are possible with this design. For higher velocities a nozzle is indispensable. Introduction of the latter or of liquid propellant causes considerable dead weight. Hohmann investigates flights in which a rocket undergoes accelerations of 15, 20, 25, 30, 40, 50, 100 and 200 m/sec^2.

He formulates the basic rocket motion equation in the same form as Esnault-Pelterie and in addition takes into account the gravitational effect.

For a manned rocket flight he assumes an acceleration of about 30 m/sec^2.

The deceleration of the rocket in the course of the takeoff, caused by the air drag, is expressed by Hohmann by the following formula (see equation (12)):

$$b_0 = \frac{R}{M} = \frac{\delta v^2}{g} \cdot \frac{F}{M}\, i.$$

He also assumes

$$\frac{F}{M} = \text{const} = \frac{1}{600}\ \frac{m^2}{kg/sec^2};\quad i^2 = 0.12;\quad g = 10.$$

Further, at altitudes beyond 50 km and velocities obtained there, the
57 aerodynamic drag may be neglected. Then at an average velocity value of 780 m/sec and a mean specific weight of air 0.2 kg/m^3

$$b_0 = \frac{0.2780^2}{10 \cdot 600} \cdot 0.12 = 2.4\ m/sec^2.$$

In Table 12, Hohmann presents the results of his calculations at different jet velocities and several assumed accelerations. The data are computed for a launch influenced by the Earth's gravity and atmospheric resistance for different flight velocities.

TABLE 12. Initial-to-final rocket mass ratios

Flight acceleration of the rocket, m/sec^2		20	100	200
Flight velocity, m/sec		9,680	10,650	10,891
Mass ratios at different nozzle velocities	2,000 m/sec	933	468	602
	2,500 m/sec	235	138	166
	3,000 m/sec	95	60	71
	4,000 m/sec	30	22	25
	5,000 m/sec		12	13

While decelerating during the descent back to Earth by means of retro jet reaction, the initial-to-final mass ratios equal approximately the squares of the numerical values given in Table 12.

Further, Hohmann investigates different rocket flight cases in interplanetary space. He introduces the calculation of the weight of a spacecraft carrying two astronauts.

He also suggests equipping the cabin with retractable wings in order to facilitate gliding upon the return to the Earth's atmosphere. According to him, landing should be performed with a parachute.

I will introduce the results of Hohmann's calculations for a rocket launched up to an altitude of 800,000 km. When this altitude is attained, a certain quantity of propellant will be necessary to change the trajectory into an elliptic one, pointed back at the Earth.

Thus a journey into space and return along an elliptic path will be carried out as follows (jet velocity 2,000 m/sec):

58 1) A 30 m/sec^2-acceleration of the missile up to a 8,490 km altitude in 8 minutes.

2) Flight up to the extreme point of the trajectory – 800,000 km. Duration 349 hours.

3) Return along an elliptic path to an altitude of 6,455 km (beginning of deceleration) – 354 hours. The total flight duration is about one month.

The rocket weight for this flight must be:

Two astronauts	200 kg
Provisions	240
Kerosene for heating	60
Oxygen for breathing and kerosene burning	200
Tanks for liquid oxygen	140
Wings and control surfaces	240
Outer shell of the rocket	780
Propellant for flight corrections on the way back	200
The same for ascent	740
Total	3,000 kg

The initial weight of the spacecraft will be 933 times this total value (see Table 12), i. e., 2,799 tons. If the propellant's specific weight is 1.5 ton/m^3, the rocket dimensions (Figure 3) will be: total height 37.58 m, base diameter 22 m, height of head 5.38 m, frontal section diameter 1.6 m, base diameter of the head 0.77 m.

Results of Hohmann's calculations for different cases of rocket flight into space are presented in Table 13.

e) Valier's work

Valier, elaborating on Oberth's studies in his investigations, arrives at the following conclusions:

With an assumed jet velocity of 4,000 m/sec and final rocket velocity of 19 km/sec, the initial-to-final mass ratio for Oberth's hydrogen rocket is about 43.1. A craft with such a performance can reach the limits of the solar system, whereas to reach just the limits of the Earth's gravitational field a ratio of 12.1 will suffice.

A flight to Jupiter and descent onto it will require a flight velocity 172 times higher than the jet velocity, and the initial-to-final mass ratio must be 4.7 trillions [10^{15}]. A journey from Earth to Jupiter,

TABLE 13

Route	Permissible acceleration, m/sec	Duration of flight	Initial weight of projectile head, tons	Initial weight of rocket in tons at different nozzle velocities, km/sec				
				2	2.5	3	4	5
Ascent from Earth prior to attainment of escape velocity, ignoring air drag	15	1192 sec	1	7,570	1,270	388	87.3	35.7
	20	762 "		2,010	438	159	44.8	29.9
	25	565 "		1,160	282	110	34.1	16.7
	30	448 "		825	216	88	28.7	14.6
	40	319 "		587	164	70	24.2	12.8
	50	248 "		495	143	62	22.2	11.9
	100	117 "		347	108	49	18.7	10.4
	200	57 "		299	95.5	44.7	17.2	9.8
Same, taking air drag into account	30	456 "	1	933	235	95	30	15
	100	123 "		468	138	60	22	12
	200	64 "		602	166	71	25	13
Flight from Earth to an altitude of 80,000 km and return journey to Earth (circumnavigation of the Moon)	30	30 1/9 days	3	2,799	–	–	–	–
Ascent from Earth, circumnavigation of Venus* and landing on Earth	30	2.15 years		83,000	–	–	–	–
Ascent from Earth, circumnavigation of Venus** and landing on Earth		1.58 years	1	82,000	–	–	–	–
Ascent from Earth, circumnavigation of Mars, and landing on Earth	30	1.5 years	16.72	567,000	69,500	17,600	3,150	1,130
Return trip Earth – Venus	30	176 days	7	54,800	8,800	2,800	620	260
Return trip Earth – Venus with refueling	30	–	2 men with supplies	670,000,000	17,000,000	1,600,000	74,000	1,240
Flight from Earth to Mars with landing on Mars	30	265 days	9	875,000	76,500	15,000	2,200	690
Flight from Mars to Earth. Refueling on Mars. Landing on Earth	30	–	–	1,430	515	265	118	71
Flight from Earth to Moon with landing on Moon	30	15 days	2.6	8,250	1,610	555	144	64
Flight from Moon to Earth. Refueling on Moon. Landing on Earth	30	–	2.6	8.9	6.9	5.9	4.8	4.3
Flight from Earth to Moon. Landing on Moon. Fueling on Earth	30	–	2.6	28,000	4,250	1,250	890	700
Flight from Moon, circumnavigation of Venus and Mars without landing, return flight with landing on Moon	30	–	2 men with supplies	2,070	780	417	194	124
Flight from Moon to Mars with landing descent on Mars	30	–	same	3,190	860	370	136	76
Flight from Moon to Venus with landing on Venus	30	–	"	200	99	67	38	29
Return trip Moon – Mars. Landing on Mars. Fueling on Moon	30	–	"	75,000	11,800	3,600	850	360
Same, to Venus	30	–	"	290,000	36,300	9,900	1,780	680

* Circumnavigation of Venus until a favorable (close) position of Earth is established.

** Without waiting for a favorable conjunction.

circumnavigation of Jupiter and return without landing on it will necessitate only $1\frac{1}{2}$ times more propellant than the flight to the Moon.

60 Some results of rocket flight calculations carried out by the scientists mentioned are compared in Table 14.

TABLE 14

Scientist and year		Tsiolkovskii 1903	Esnault-Pelterie 1913	Goddard 1909		Oberth 1923	Valier 1924	Hohmann 1925	
Jet velocity, km/sec		5.7	65.3	3.63	2.134	3.0	4.0	5.7	2.0
Flight acceleration, m/sec^2		100	10.8	45.7	45.7	40	–	100	30
Initial-to-final mass ratio	Launch from Earth into infinity	9	–	43.5	602	200	–	10	933
	Launch from Earth and landing on Moon	9	–	–	–	–	12.1	–	8,250
	Takeoff from Earth, circumnavigation of Moon and return to Earth	< 9	–	–	–	200	–	–	933
	Launch of Earth satellite	5	–	–	–	–	–	–	–
	Launch from Earth for flight to Mars	< 9	–	–	–	–	–	–	875,000
	Launch from Earth for flight to Venus	21	–	–	–	–	–	–	54,800
	Launch from Earth and travel to another solar system	21	–	–	–	–	43.1	–	–
	Takeoff from Earth, circumnavigation of Mars and return to Earth	10,000	–	–	–	–	–	–	–
	Takeoff from Earth, descent onto Moon, return to Earth using retro jet thrust	–	1.43	–	–	–	–	–	–

10. SIDEREAL NAVIGATION

Questions of orientation in space and sidereal navigation during interplanetary travel have been investigated very little so far. The following problems will have to be solved:

1) Determination of the velocity of flight.
2) Determination of the acceleration of flight.
3) Determination of the spacecraft position relative to the planets.
61 4) Determination of the position of meteorites encountered.
5) Determination of the limit of a planet's atmosphere.
6) Determination of temperature in interplanetary space.
7) Methods of observation of outer space.
8) Determination of solar radiation.
9) Determination of the influence of meteorites, etc.
10) Determination of flight-direction variations.
11) Flight stability.
12) Controllability.

Knowing the flight acceleration and time at different moments, one can find the flight velocity. Time will have to be measured by spring watches, acceleration by devices based on the principle of inertia. The rocket's position in space can be determined by observing different stars and planets and taking into account its velocity and flight time. Approaching meteorites may be detected visually or by special instruments yet to be invented. Collision of a spacecraft with a meteorite is very improbable. Entry into an atmosphere may be detected by an increase in heat of the rocket skin or instruments, or it may be detected visually. Temperature and solar radiation in space will be measured with special instruments. Observation of the outer world can be performed with periscopes and special reinforced transparent glass. The influence of meteorites on rocket course will be corrected by engine bursts. Flight direction will also be changed by bursts. Stability is maintained by bursts or by gyroscopies in combination with moving masses inside the rocket.

11. CONCLUSIONS

The discussion of the results of the investigations carried out by different scientists leads to the following conclusions:

1. Taking off from the Earth and reaching considerable altitudes is possible, and will be more easily attained, the higher the acceleration imparted to the rocket.

2. Launch of a manned spacecraft is much more difficult to realize because of acceleration limitations (not higher than 4g) during the ascent and the descent.

3. Descent of a manned craft onto the Earth with the aid of retrorockets is not advantageous. It seems that a combined landing will have to be adopted. The re-entry maneuver will begin with circling the Earth in elliptic orbits. These orbits will gradually become circular (due to the decelerating effect of the atmospheric resistance) and then the last stages of gliding and finally parachute-landing will take place. The scheme in Figure 22 presents a launch with such a descent on re-entry, after Hohmann. The Earth on the figure is hatched and the atmospheric limits are indicated by the dashed circle. The rocket takes off from (a) and

62 climbs up to (b). It follows an ellipse on its way back and re-enters the terrestrial atmosphere at point (c). The air resistance decelerates the craft and gradually shortens the elliptic orbit, converting it into a circle (d). The rocket glides farther down to (e) and finally performs a parachute landing.

4. It is easier to realize a space flight with a higher ejection velocity of the combustion products.

5. An inclined launching is much more advantageous than a vertical one.

6. In general, the problem of interplanetary flight can be solved. But to realize even a lunar circumnavigation it is necessary to carry out a series of experiments and investigations, which will demand considerable resources.

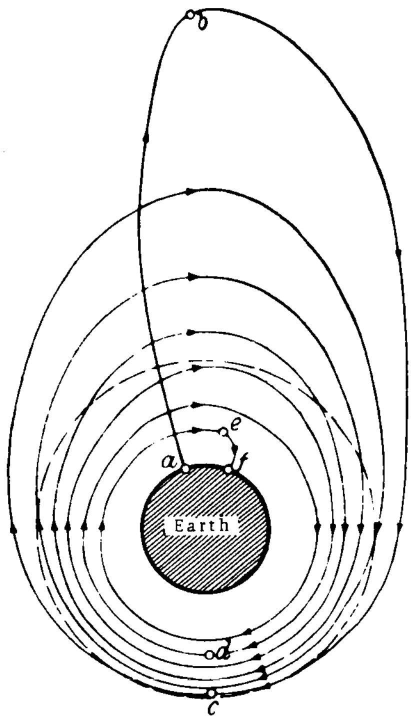


FIGURE 22. Take-off and descent of a rocket, after Hohmann

12. LITERATURE ON INTERPLANETARY FLIGHT

Numerous publications in the field of interplanetary flight have recently appeared. Many works deal with specific aspects of the general problem. I have collected about 300 references in Russian and about 200 in other languages.

Here I limit myself to a list of the most recent and important scientific works (in chronological order) on the subject:

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* [Available in English translation, TT70-50111, NASA TTF-640.]
** [Available in English translation, TT70-50112, NASA TTF-641.]
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