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Chapter 2

The Origin of Gravity-Propelled Interplanetary Space Travel*

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Introduction

Using the gravitational influence of an easy-to-reach nearby planet to catapult a free-fall space vehicle to a more distant planet, that would ordinarily require substantial rocket propulsion using traditional direct-transfer trajectories, has had a significant effect on the methods and pace of planetary exploration. Moreover, by utilizing a series of such trajectory-changing planetary encounters, it is possible for a free-fall vehicle to travel to many different planets without requiring any onboard rocket propulsion. This innovation, which is usually called "gravity-assisted" or "swing-by" trajectory, represented the key propulsion breakthrough that opened up the entire Solar System to exploration using relatively small, chemically propelled launch vehicles. The Mariner 10 Earth-

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Venus-Mercury Spacecraft, the Pioneer 10 and 11, the Voyager 1 and 2, the low launch energy Galileo mission to Jupiter, and the Ulysses mission to the Sun were made possible by this innovation. This "gravity-propulsion" concept originated in 1961 in the mind of an American, Dr. Michael A. Minovitch, then a graduate mathematics student from the University of California, who had no prior experience in astrodynamics. It represented a complete break with traditional ideas about space travel.

Prior to this innovation, it was taken for granted that the rocket engine represented the basic, and for all practical purposes, the only means for propelling a space vehicle through the Solar System. Access to the planets was restricted by the fundamental limitations of chemical rocket propulsion. The famous Hohmann Trajectory was universally accepted as the minimum energy and, thus, the optimal path for traveling to the planets. But using this minimum energy trajectory for traveling to the most distant planets requires extremely long trip times—another limitation. Leading astrodynamicists suggested a variety of large and exotic propulsion systems to overcome these obstacles and extend a space vehicle's reach—but not by far, and by no means cheaply.

Meanwhile, leading theorists continued to struggle with the adaptation and refinement of special trajectory problem formulations, such as the relatively straightforward Two-Body Problem and the quite difficult Restricted Three-Body Problem. Many researchers in the 1950s were attempting to apply new digital computing techniques to these problems, with mixed results. Concurrently, G. Crocco, Harry O. Ruppe, D. F. Lawden, Krafft Ehricke, R. H. Battin and others investigated the complicating effects of planetary gravitational perturbations. They extended the early work of Yu. V. Kondratyuk and A. F. Tsander from Russia and Walter Hohmann from Germany, but they reached the same basic conclusion that Hohmann's trajectory was still the minimum-energy trajectory for traveling to the planets, and they supported this belief with numerous mathematical demonstrations. Planetary gravitational perturbations were generally regarded as annoying disturbances, and the common method for dealing with them was to resist their effects—often by onboard rocket propulsion. Conquering nature by brute force—huge launch vehicles equipped with exotic nuclear upper stages— seemed to be the only way to reach the most distant planets.

Using elegant vector techniques that he developed for studying three-dimensional astrodynamic problems, Minovitch combined them with the computational power of a digital computer to obtain the first numerical solution to the famous unsolved Restricted Three-Body Problem. This vector formulation removed many of the complicated scalar equations and enabled him to focus on the underlying physics of the problem. Minovitch discovered that a large amount of orbital energy can be interchanged between a free-fall interplanetary space vehicle and a passing planet via gravitational interactions, and that these effects can be used serially to propel the vehicle around the entire Solar System indefinitely, with radical trajectory changes relative to the Sun, without any rocket propulsion. This gravity propulsion concept represented a quantum leap in space travel.

This paper is the first in a series describing the details of this discovery, Minovitch's early work in developing it, and showing how the various NASA gravity-propelled missions originated from it.

The concept of gravity-propelled interplanetary space travel—referred to herein as gravity propulsion—involves launching a space vehicle on a transfer trajectory (typically a low-energy, near-Hohmann path) to some nearby initial planet, and then using the gravitational influence of that planet to catapult the vehicle to a more distant planet or region, without rocket propulsion to reduce the launch energy ordinarily required to reach the distant planet, using a conventional direct-transfer Hohmann trajectory. By utilizing a series of such planetary encounters, it is possible, in principle, to propel a vehicle from planet to planet anywhere around the Solar System.

After the vehicle is injected onto the initial transfer trajectory to the first planet on its tour, no further rocket propulsion is required to effect major trajectory changes. In the theoretical formulation of this concept, onboard vehicle propulsion was assumed to be absent or inhibited, and thus the vehicle was considered to be on a free-fall trajectory at all times, flinging from planet to planet. In practice, some onboard propulsion is required for guidance to perform minor course corrections and perhaps attitude control functions.

Physically, the essence of gravity propulsion is that the space vehicle and the planet exchange orbital energy during their encounter—the planet loses a little and the vehicle gains the same amount, or vice versa. Which body gains or loses depends on the geometry of the encounter. During these encounters, the vehicle's trajectory can be radically bent by the gravitational influence of the planet; such paths are commonly called "gravity-assisted" or "swing-by" trajectories. Indeed, the change in the vehicle's velocity with respect to the Sun—one key effect of gravity propulsion—is directly proportional to the magnitude of this bending angle. The force F acting on the vehicle follows from Newton's Law of Gravity

$$F = G\frac{m_1 m_2}{r^2} \qquad (1)$$

where m_1 and m_2 denote the mass of the planet and vehicle respectively, r_2 is the distance between their centers of mass, and G is equal to the universal gravitational constant. Thus, the more massive the vehicle, the larger the propulsive force.

The development of the gravity propulsion concept represents one of the most significant advances in the field of astronautics. It enables the exploration of any planet or region in the Solar System with relatively small launch vehicles and conventional chemical rocket propulsion, primarily because it significantly reduces the launch energy requirements of most missions. Gravity propulsion employs intermediate planets as a propulsive means for getting to target planets, greatly increasing available launch windows for many types of ambitious missions. Consequently, it offers the possibility of exploring many different planets with the same vehicle. Finally, it enables missions with much shorter trip times than comparable missions using chemical propulsion and conventional direct-transfer Hohmann trajectories.

The Foundation of Astronautics Prior to 1961

Prior to the innovation of gravity-propelled trajectories, it was taken for granted that the rocket engine, operating on the well-known reaction principle of Newton's Third Law of Motion, represented the basic, and for all practical purposes, the only means for propelling an interplanetary space vehicle through the Solar System. In fact, this traditional concept was taken as one of the most fundamental axioms of space travel. As a result, only a relatively small portion of the Solar System near the ecliptic plane could be explored by space vehicles using conventional chemical rocket propulsion. More distant regions, out to the orbits of Neptune and Pluto, for example, would require many decades of travel time and were therefore considered inaccessible with chemical rocket propulsion.

It was also assumed (and proven mathematically) that the minimum-energy transfer trajectory to another planet was an elliptical path tangent to the launch planet's orbit at departure and tangent to the target planet's orbit at arrival. This ideal "optimal trajectory," illustrated in Figure 1, became known as the "Hohmann Trajectory," in honor of Walter Hohmann, a German architect who discovered these trajectories in 1925. Prior to the innovation of gravity-propelled trajectories, this minimum-energy transfer trajectory represented another fundamental axiom of space travel.[1-19]

A Hohmann Earth-Neptune trajectory illustrated in Figure 2, which requires a one-way trip time of about 31 years, highlights the main disadvantage of Hohmann trajectories: they require very long trip times (in fact, they result in maximum trip times when the heliocentric transfer angle is less than or equal to 180°).

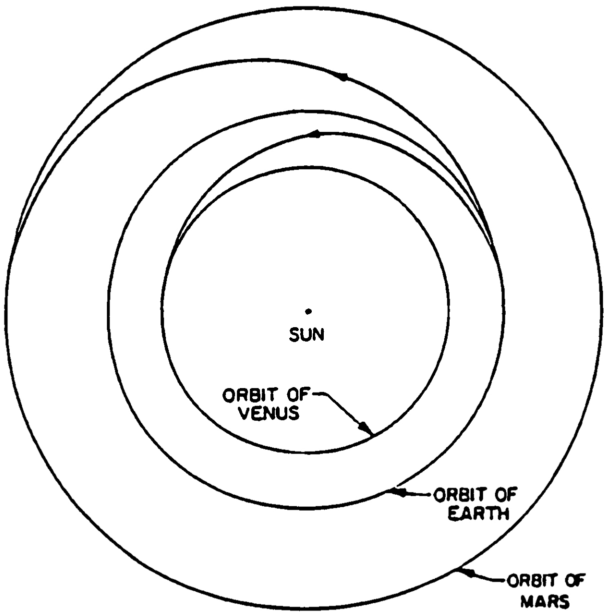

Figure 1 Classical Hohmann minimum-energy trajectories from Earth to Venus and Mars.

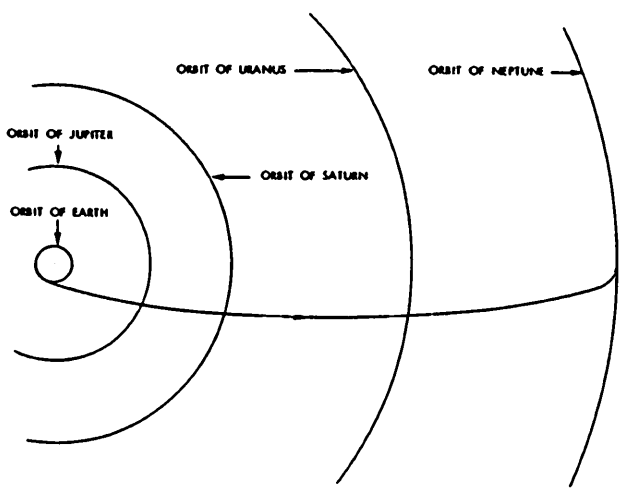

Figure 2 Hohmann minimum-energy trajectory to Neptune (trip time = 31 years, V_∞ = 12 km/sec).

Regions of the Solar System near the Sun and out of the ecliptic plane required so much launch energy that they were considered to be impossible to reach with conventional chemical rocket propulsion. This holds even when using enormous launch vehicles, such as the Novas, Super Novas and Sea Dragons designed during the early 1960s (Figure 3). It was believed that the only practical method for propelling a space vehicle into these hard-to-reach regions (which represented over 95% of the Solar System) was by means of very exotic propulsion systems, such as electric propulsion.[20-29] All of these advanced propulsion systems were based on the traditional reaction principle.

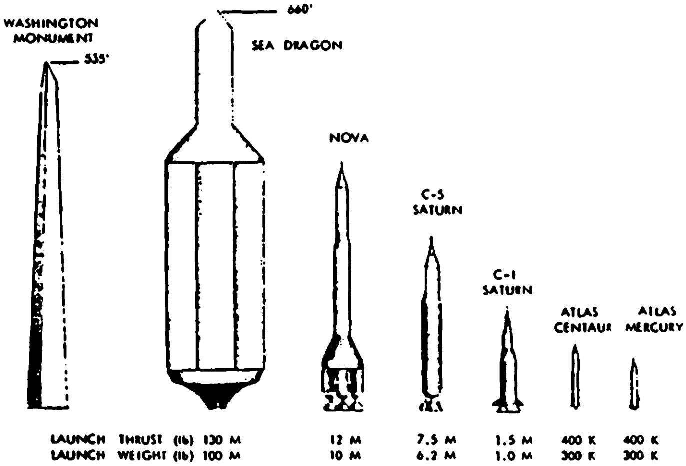

Figure 3 Proposed launch vehicle designs of the early 1960s based on Hohmann trajectories and brute-force rocket power (from Reference 32).

This, in brief, was the technical foundation of astronautics upon which the plans for the United States' interplanetary space program were based at the beginning of the 1960s.[30-48] The Soviet Union was operating under a quite similar theoretical foundation, though technically its program was based on more capable launch vehicles. Since this foundation was itself based on decades of research,[49] there was no expectation that anything would change.

Of the many considerations involved in interplanetary space travel, trajectory design is the most important. Trajectories determine launch velocities, and launch velocities, together with vehicle mass, determine launch energies. Trajectories also determine the launch windows required for traveling to other planets. Trajectory determination and analysis is a process that relies heavily on theoretical knowledge and procedures. This body of knowledge rests upon the principles of celestial mechanics, and ultimately upon a branch of theoretical physics called analytical mechanics.

Since the mass of the Sun is so much greater than any planet, the trajectory of a free-fall vehicle moving through interplanetary space is essentially an exact conic path, unless it is close to a perturbing planet. This is because the solution of the differential equations of motion of a body moving under the gravitational influence of a single massive body is a conic section, such as an ellipse. Thus, the orbits of all the planets moving around the Sun are essentially constant elliptical paths; they represent the solution trajectories of the "Two-Body Problem."

Since the orbits of most of the planets in the Solar System are nearly circular and co-planar, Hohmann and other early astrodynamicists incorporated this fact into their analysis of rocket-propelled interplanetary trajectories.[50] They regarded an interplanetary trajectory as an arc of a constant elliptical path that is co-planar with the orbits of the launch planet and the target planet. This co-planar, two-body assumption was maintained for several decades as standard practice in determining and analyzing interplanetary trajectories.

Many early investigators of space travel recognized that a non-stop, free-fall, round-trip interplanetary trajectory to Mars or Venus would be useful as a preliminary manned planetary reconnaissance mission, before undertaking an actual landing mission. Significant efforts and resources were expended in search of these appealing cases. These trajectories were designed as constant elliptical paths, and the goal was to ensure that when the vehicle approached the orbit of the target planet, the planet would be relatively close; and when the vehicle returned to Earth's orbit, the Earth also would be relatively close.

Unfortunately, since the gravitational perturbations of the target planet make a constant elliptical path impossible (the trajectory would depart from its "ideal" elliptical path), the Earth would not be very close to the vehicle upon its return. Thus, planetary gravitational perturbations were generally viewed as annoying disturbances that destroyed the constant elliptical paths so carefully calculated by trajectory designers to produce the required planetary interceptions.

To circumvent this problem, Hohmann canceled out the perturbations caused by the target planet by using onboard rocket propulsion (see pages 79-81 of Reference 50). The propulsive thrust of the vehicle's rocket engine was designed to be equal in magnitude but opposite in direction to the planet. This method of canceling out planetary perturbations by using onboard rocket propulsion was adopted by other astrodynamicists and used up to the 1960s (see p. 1062 of Reference 7).

Another method suggested was simply not to allow the trajectory to pass very close to the target planet (see page 1-32 of Reference 51). But since this method degraded the viewing of the target planet, it was not very practical. (The objectives of non-stop round-trip missions called for a close pass by the target planet to maximize the observational opportunities.)

There is a third possibility for circumventing this perturbation problem. In principle, the approach trajectory could be designed to ensure that the planet's gravitational influence would change the return leg of the round-trip trajectory, such that the vehicle returns to Earth automatically. Since this method does not require the application of rocket propulsion to cancel out the perturbations, and since it allows the vehicle to pass arbitrarily close to the target planet for good viewing, one might ask why astrodynamicists did not automatically choose this method over the other two. One fundamental reason is that designing such a trajectory for an actual mission required a numerical technique for solving one of the most difficult problems in Celestial Mechanics—the Restricted Three-Body Problem.[52-55]

The Restricted Three-Body Problem involves finding the trajectory of a body of negligible mass (such as a free-fall space vehicle), moving in a plane under the simultaneous influence of two other bodies (Earth and Moon, for example), that are themselves moving in the same plane under their mutual gravitational influence. The problem is difficult, because there exists no closed-form solution for it (i.e., the differential equations of motion are non-integrable). From any given initial position and velocity, there is no general formula for determining the position of the vehicle at any future time, which is exactly what trajectory and mission designers want to know. The non-existence of closed-form solutions was proven by Poincaré near the end of the 19th century.[56]

In the case of the Two-Body Problem (e.g., Earth and Moon alone, or Earth and spacecraft alone), the trajectory of each body moving under the mutual gravitational influences is exactly expressible as a simple quadratic function that defines a conic section. But once a third body is introduced (the Sun, for example), the trajectory of each body is extremely difficult to determine precisely.

Figure 4 illustrates this problem in the case of round-trip free-fall interplanetary trajectories. When a free-fall vehicle is moving close to the target planet along a path AB, its motion is influenced by the gravitational fields of the planet and the Sun acting simultaneously. The planet is moving with an essentially constant velocity V_p in its orbit around the Sun.

The essence of the Restricted Three-Body Problem involves precisely determining the vehicle's position and velocity vectors $\vec{r}, \vec{v}$ with respect to the perturbing planet as the vehicle passes by. For round-trips from Earth (shown in Figure 4), the problem is to calculate the vehicle's trajectory as it approaches the planet at point A, passes around the planet along the path AB, and moves away along interplanetary trajectory BC, which intercepts the Earth at point C. Finding the precise path ABC proves to be quite difficult.

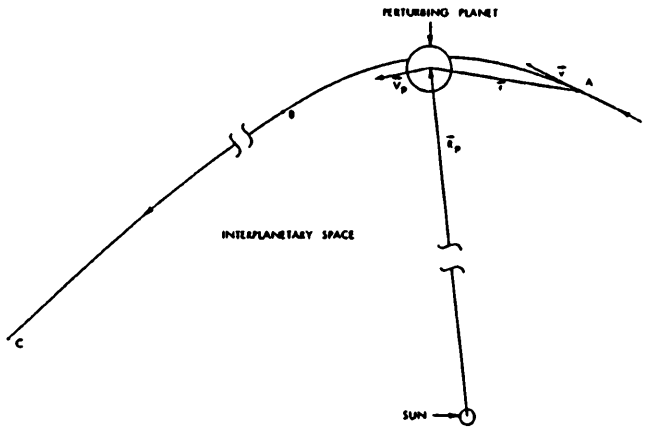

Figure 4 Elements of the three-dimensional Restricted Three-Body Problem.

At the beginning of the 1960s, there was no general numerical solution for precisely determining this path (i.e., for determining the vectors $\vec{r}$ *and* $\vec{v}$ at point A). The theoretical foundation of celestial mechanics could not circumvent this problem. But if one studies the technical development of space travel between 1925 and the early 1960s, it becomes clear that this theoretical impasse was not regarded as a serious impediment. The engineering of space travel was directed at developing more powerful and exotic rocket propulsion systems—all based upon the reaction principle—in an attempt to conquer nature by brute force rocket power.[38-40] This was the general technical methodology for achieving interplanetary space travel.

Although the general Restricted Three-Body Problem was unsolvable up to that point in time, the advent of high-speed digital computers in the 1950s introduced a means that, in theory, could be used to obtain numerical (i.e., computer-based) solutions to it with any desired accuracy. Since the differential equations of motion were known, the exact numerical solution of the entire trajectory ABC could, in principle, be obtained by a numerical integration and iteration process known as "differential corrections."[57-59] This development can be regarded as an important technical achievement that enabled significant theoretical advances in astronautics (and applied mathematics in general).

Since in principle the gravitational influences of all the major bodies in the Solar System could be simultaneously incorporated into this algorithm, the result could be more accurate than any theoretical solution of the Restricted Three-Body Problem. Thus, by utilizing the sheer computational power of high-speed electronic digital computers, the numerical solution of the Restricted Three-Body Problem (or N-body Problem) was reduced to that of finding an analytic method for determining a sufficiently accurate three-dimensional initial approximation (an intermediate orbit, i.e., the position and velocity vectors at point A in Figure 4), that would converge to the exact solution during the numerical integration/iteration process. Fed the proper inputs, the computer would do all of the hard, computation-intensive work.

Despite many determined efforts, at the beginning of the 1960s, a general numerical solution of the Restricted Three-Body Problem (using computers and initial approximations) did not exist. No one—American, Soviet, or European—could solve it. But many tried.

One well-known and generally accepted approximate solution to the Restricted Three-Body Problem used by astrodynamicists (in the early 1960s) could be obtained by assuming that the perturbing minor body (planet) is fixed relative to the major body (Sun). This simplification (due to Euler) is called the "Two Fixed Force-Center Problem[60] (See also Sec. 203, page 145, Reference 53). Since the space vehicle spends a relatively short time in the vicinity of the perturbing planet compared to the planet's orbital period, this assumption appeared to be a valid approximation of the real situation. As Euler discovered, this simplification led to differential equations of motion that are integrable. Hence, the approximate trajectory was expressible in closed-form (in terms of elliptic functions). Although this simplification did enable the determination of exact numerical solutions for round-trip lunar trajectories, it was not successful in determining exact round-trip interplanetary trajectories; the latter are far more difficult.[61]

Essentially, this is due to the basic geometry of the situation: the interplanetary legs extend over distances many times greater than the arc AB in the vicinity of the perturbing planet. Thus, unless the initial approximation of the trajectory around the perturbing planet can be determined fairly accurately, (i.e., the position and velocity vectors at point A in Figure 4), a numerical integration/iteration process will not converge to the exact solution. The situation is highly nonlinear and can be regarded as a mathematical example of chaos.[62,63] Another important geometric complication in the interplanetary case was the fact that the Restricted Three-Body Problem (and Euler's simpler model) assumed, trajectories were confined to a single plane, but the planets do not follow this assumption. Applying this two-dimensional assumption to Earth-Moon round-trip trajectories works well, but applying it to interplanetary trajectories leads to failure. Consequently, the reason there was no general numerical method for determining precise interplanetary round-trip trajectories by the early 1960s was, because no analytic method existed that would generate a sufficiently accurate initial approximation of a spacecraft's approach trajectory at a perturbing target planet. Without this approximation, the state-of-the-art numerical techniques could not converge.

There is one particular consequence of Euler's Two Fixed Force-Center Problem that should be emphasized. If the perturbing minor body is assumed to be stationary with respect to the major body, then the orbital energy of a free-fall vehicle relative to the major body cannot be changed by passing close to the minor body, because orbital energy in this frame of reference is an exact invariant and must therefore remain constant (see Sec. 5, page 88 of Reference 60). In this theoretical framework, a spacecraft passing Mars, for example, would have the same orbital energy with respect to the Sun after Mars's closest approach as it had before; at points along its trajectory of the same given radius from Mars before and after closest approach (points A and B in Figure 4, for example), it would have the same velocity with respect to the Sun. Likewise, the orbital energy of Mars and its heliocentric velocity would remain at their pre-encounter values.

The basic concept of gravity propulsion—which is contrary to this conclusion—could not have been discovered within this basic theoretical framework of the Two Fixed Force-Center Problem of celestial mechanics. By the 1950s and early 1960s, the validity of this framework for determining approximate solutions to selected cases (primarily lunar) of the Restricted Three-Body Problem was a strong motivator for investigating the properties of motion in interplanetary space using the same technique.[64] Thus, when the concept of gravity-propelled interplanetary space travel (Minovitch hadn't named it gravity propulsion yet) was originally proposed by Minovitch in 1961, there were more than a few astrodynamicists who evidently believed such a concept was theoretically impossible, because it would violate the law of conservation of energy.

In 1959, Richard Battin developed the first mathematical technique for determining three-dimensional conic approximations for the interplanetary legs of round-trip free-fall trajectories to Mars.[65] Basically, the method involved patching two elliptical trajectories together at the center of Mars, that had the same approach and departing velocities relative to Mars, and that began and ended at Earth. As was common practice at that time, Battin treated the perturbation of Mars as occurring instantaneously when the vehicle crossed the Martian orbit (page 557, Reference 65).

Although Battin was able to determine the eccentricity and semi-major axis of a hyperbolic conic approximation of the encounter trajectory AB (refer to Figure 4), a detailed three-dimensional determination of the encounter trajectory (with six orbital elements) was not found. But, as discussed above, the encounter arc AB is the most important part of a round-trip free-fall trajectory, and a complete initial approximation of it is required before an exact determination of the entire trajectory can be obtained by differential correction numerical methods. However, Battin was directing his efforts primarily at determining approximate trip times, launch energies and distances of closest approach for initial studies of three-dimensional round-trip missions to Mars. His work was not explicitly targeted at formulating a numerical method for solving the Restricted Three-Body Problem.

Battin also believed that the orbital energy of an interplanetary free-fall trajectory should remain constant after passing a perturbing planet. This is supported by a description of his own numerical trajectory calculations, where he indicates a change in orbital energy as a "curious fact" (paragraph 2, page 566, Reference 65).

It should also be emphasized here that the problem of determining a gravity-propelled trajectory involves solving the Restricted Three-Body Problem, which had never been done prior to 1961. This is evident from Figure 4, where the arc can be regarded as the gravity-propelled portion of such an interplanetary trajectory (elliptical or hyperbolic), which catapults a free-fall vehicle without rocket propulsion to a more distant third planet represented by point C (which is not the launch planet). Point C could also represent some planet in a series of gravity-propelled encounters. As illustrated in Figure 4, the two massive bodies (Sun and planet) and one negligible-mass body (space vehicle) interact as assumed in the formulation of the Restricted Three-Body Problem.

Thus, in the absence of a theoretical solution, investigations into the nature of the gravity propulsion concept require a numerical solution of the Restricted Three-Body Problem—an analytic method. It is clear, then, that this concept could not have been explicitly formulated prior to obtaining this solution—the three-body interactions are simply too complex and subtle to characterize and quantify intuitively or by "back-of-the-envelope" means. And for mission design purposes very precise trajectory solutions are required. The solution of this problem was not available at the start of the 1960s and, moreover, it was regarded by astrodynamicists to be among the most difficult problems in celestial mechanics.

As the previous discussion shows, a general propulsion concept that involves propelling a vehicle around the Solar System with little or no rocket propulsion, via planetary gravitational forces, was not even close to realization in the minds of the professional astrodynamicists at the dawn of the 1960s. On the contrary, since orbital energy was regarded by many as an invariant of motion, just the opposite was true. To be sure, many leaders in the field were fully aware of the reality of planetary perturbations, but aside from their possible use in round-trip interplanetary trajectories, no one proposed using them as a basic substitute for onboard rocket propulsion. In particular, no one suggested that the launch energy associated with Hohmann's direct transfer trajectory to a distant target planet could be reduced by launching a vehicle to some easy-to-reach nearby intermediate planet, and letting the gravitational forces of that intermediate planet propel the vehicle to the distant target planet instead of using onboard rocket propulsion. Since this possibility was not proposed prior to 1961, the more radical possibility of using planetary perturbations serially by multiple encounters to achieve unlimited interplanetary propulsive mobility throughout the Solar System was even more remote. Planetary perturbations were generally regarded as annoying disturbances of two-body trajectories that made the mathematics of high-precision trajectory determination very complicated.[59]

This, then, was the underlying technical and theoretical foundation of interplanetary space travel prior to the introduction of gravity propulsion.

Discovering Gravity-Propelled Interplanetary Space Travel

The following discussion is a detailed account of events that occurred at the Jet Propulsion Laboratory (Pasadena, California) and the University of California at Los Angeles (UCLA), between the Spring of 1961 and early 1962. All dates are for 1961, unless noted otherwise. Numerous references to source material are included to support the text. Direct quotations from Dr. Minovitch are from a series of recorded interviews conducted during the Spring of 1990.

In early Spring of 1961, Michael Minovitch decided to seek temporary summer employment at the California Institute of Technology's Jet Propulsion Laboratory (JPL). At that time, he was a full-time, third-year graduate student at UCLA, working toward Ph.D. degrees in both mathematics and physics. He had worked the previous summer in Dr. Linus Pauling's laboratory at CalTech, determining the structure of crystals from X-ray diffraction data and Laue photographs, and he had become aware of JPL while on campus.

Minovitch was initially offered summer employment in two different groups in JPL's Systems Analysis Section (Section 312); one was the Trajectory Group, headed by Mr. Victor Clarke, Jr., and the other was a theoretical group, headed by Dr. William Melbourne. Minovitch selected Melbourne's group because it was more theoretical, and because he had never worked in the field of trajectory determination and had never taken any courses in astrodynamics or celestial mechanics.[66]

When he arrived at JPL in June, Minovitch was told of a change in plans by Tom Hamilton (the person who interviewed him for possible summer employment), because Clarke was having some problems in calculating one-way interplanetary conic trajectories for NASA's Mariner program. In 1960, Clarke had formulated a method for calculating such trajectories numerically on JPL's large IBM 7090 digital computer, but the solution had some numerical non-converging iteration problems inherent in the analytic formulation.[67] Under certain conditions, the semi-major axis of a conic trajectory failed to converge in an iteration process, because the initial approximation at the beginning of the iteration was too far from the actual value. Although Minovitch had no experience with interplanetary trajectories, he agreed to work on the problem in Clarke's group, instead of working on more theoretical problems in Melbourne's group.

The assignment from Clarke involved formulating an alternative method for determining the semi-major axis and eccentricity of a conic trajectory passing between two given points with a prescribed trip time.[68] It was a very clear, precise and unambiguous assignment. Minovitch formulated a solution using Lambert's equations and documented his work as an internal JPL Technical Memorandum, dated July 11.[69] Clarke was very satisfied with the paper and instructed Minovitch to make sure the solution was free of errors and to prepare the memorandum as an external JPL Technical Report. Some relatively minor errors were uncovered that were corrected in an Errata dated August 29.[70]

While checking his memorandum for errors and preparing it for publication, Minovitch noticed that the underlying mathematical techniques then used to define and analyze three-dimensional interplanetary trajectories were basically those developed many decades ago by mathematical astronomers for determining the orbits of various celestial bodies.[69] The orbit or trajectory was described by a set of six "orbital elements:" semi-major axis a, eccentricity e, inclination i, longitude of ascending node Ω, argument of periapse ω, and time of perifocal passage T (see, for example, Reference 54). Although this orbit representation was satisfactory for astronomical purposes, it results in very long and complicated mathematical equations in astrodynamic problems. By this time, Minovitch knew that velocity vectors were very important in astrodynamic problems, and he noted that in the six-element formulation it was cumbersome to express such vectors mathematically in three-dimensional space. According to Minovitch:

> I loved mathematics and theoretical physics; I had a passion for these subjects and for their inherent esthetic beauty. When I saw how messy the six-element formulation was, I decided to see if I could clean it up. I began, on my own initiative, to investigate the possibility of representing a conic trajectory in three-dimensional space as a set of two constant orthogonal three-dimensional vectors, instead of six independent scalars as JPL was doing. I wanted to use this vector representation to express a vehicle's ve-

locity vector at any point on its orbit by a simple compact vector equation. Why? Because it was simpler, and it looked better . . . elegant, you might say.

Essentially, this research was purely theoretical, and it involved developing vector equations for determining three-dimensional conic orbits satisfying various boundary conditions, and for the inverse problem of determining various vector properties of a given conic orbit from its constant orbital vectors. At that time (1961), vector analysis was a relatively new mathematical tool in astrodynamics and celestial mechanics.[71] But with a strong mathematical background, it took only a few days for Minovitch to build the mathematical techniques he sought, and shortly thereafter he started looking for a challenging problem to use them on.

The problem Minovitch selected to investigate with his newly developed vector methods was the Restricted Three-Body Problem (in three-dimensions), represented by a round-trip free-fall trajectory to some target planet. As pointed out above, Battin had also investigated round-trip free-fall trajectories, but he used the traditional six-element representation, which resulted in complicated mathematical expressions.[65] Moreover, Battin did not completely determine the critical hyperbolic encounter trajectory AB shown in Figure 4.

Having taken a course in the numerical solution of simultaneous differential equations as an undergraduate, Minovitch recognized that a numerical solution of the unsolved Restricted Three-Body Problem could, in principle, be obtained by first determining a sufficiently accurate initial approximation of the encounter trajectory AB (represented by the position and velocity vectors $\vec{r}$ *and* $\vec{v}$ at point A in Figure 4), and by applying the methods of integration/iteration differential corrections.

Minovitch also recognized that the time spent in the vicinity of the perturbing planet is relatively small, so that if the gravitational field of the planet were neglected, the path AB relative to the Sun would be very nearly equal to a straight line. Thus, when the vehicle is close to the planet, it can be assumed that only the gravitational field of the planet influences its motion. When the vehicle is far from the planet, it can be assumed that only the Sun influences the vehicle's motion.

Minovitch developed initial criteria to determine the size of this planetary gravitational influence region based upon the geometrical properties of hyperbolic asymptotes. However, while reading some of Poincaré's works on celestial mechanics, he came across a paper by Tisserand, describing a more practical method for determining the size of this "sphere of influence."[56,72]

By assuming that only the planet or the Sun influences the vehicle's motion—depending upon whether the vehicle is moving inside or outside the planet's sphere of influence, respectively—and by using his new vector methods, Minovitch was able to determine a three-dimensional conic approximation of the encounter trajectory AB, the vehicle's precise entry point $\vec{r}$, and the corresponding velocity vector $\vec{v}$ on the sphere of influence (at point A in Figure 4). His overall conic approximation was a smooth continuous trajectory (without corners) consisting of two interplanetary legs connected to each asymptote of a hyperbolic encounter trajectory AB positioned between them (see Figure 4).

Due to the dynamics and geometry of the situation, Minovitch knew that this approximation must be fairly close to the actual trajectory, where the vehicle's motion is continuously influenced by the gravitational fields of the planet and the Sun acting simultaneously. By using methods of differential corrections, Minovitch found that the complete hyperbolic conic approximation of the encounter trajectory AB provided a sufficiently accurate initial trajectory for numerically determining the solution to the unsolved Three-Body Problem to any desired accuracy. Moreover, he realized that by "switching on" the influences of the other planets (which was straightforward in the integration process), an exact numerical solution to any desired degree of accuracy could, in principle, be obtained.

Finding a practical general numerical solution to the unsolved Restricted Three-Body Problem was a notable achievement in its own right in the field of theoretical celestial mechanics. But Minovitch's work resulted in a more significant discovery in the field of astrodynamics: gravity propulsion.

As noted earlier, Euler's approximate solution of the Restricted Three-Body Problem assumes that (1) the motion of a vehicle passing a perturbing planet is influenced by the gravitational fields of the Sun and the planet acting simultaneously (which is true), and (2) the perturbing planet is at rest relative to the Sun (which is not true and leads to the erroneous conclusion of constant orbital energy). Recall that the analytical consequences of this assumption imply that the orbital energy of the free-fall vehicle (and the orbital energy of the planet) relative to the Sun before and after the encounter must remain constant.

Minovitch's approximate solution to the Restricted Three-Body Problem assumes that (1) only the perturbing planet influences the motion of a passing vehicle (which is not true in theory but is a close approximation to the true situation), and (2) the perturbing planet is moving in its orbit relative to the Sun as in the real situation. The consequences of these assumptions are quite important. They enabled Minovitch to transform the intractable Three-Body Problem into a system of Two-Body Problems and, unlike Euler's approximate solution, revealed that the vehicle and the perturbing planet exchange orbital energy relative to the Sun via gravitational interactions during the encounter.

During this research (which was conducted at home and at JPL, without Clarke's knowledge or permission), Minovitch noticed this fundamental property of his approximate solution: the amount of orbital energy exchanged between the space vehicle and planet could be *very* large. While studying the mathematical details and the basic physics of the situation (his insight enhanced by the elegant and simplifying vector formulation), Minovitch recognized several important effects. As he recalls,

> My vector methods worked so well that they converted the challenge of understanding this problem [the Restricted Three-Body Problem] from a mathematical challenge to a physics challenge . . . one of trying to visualize what the equations were indicating. The simple transformation from a planet-centered inertial reference frame to a Sun-centered frame had important consequences. When I studied the relatively simple equations on my paper, these things just leaped out at me!

The gravitational influence of the passing planet caused the direction and magnitude of the vehicle's velocity vector to be radically changed relative to a Sun-centered reference frame—and the energy producing this change came directly from the planet's orbital energy. It was a harnessing of the planet's essentially inexhaustible orbital energy. Minovitch recognized that these effects could be utilized as a free propulsive thrust-generating mechanism for achieving major trajectory changes relative to the Sun without rocket propulsion, thereby significantly reducing the launch energy requirements for interplanetary space travel previously based upon Hohmann direct-transfer trajectories. He concluded that they could be used to propel a free-fall space vehicle passing a nearby planet to a more distant planet, and this new planet could provide the energy to catapult the vehicle to another planet, and so on, in a self-sustaining chain reaction situation that could continue indefinitely. Minovitch realized that after a vehicle reaches the initial planet (using a relatively small amount of initial onboard rocket propulsion), these chain-reaction effects constituted a new method for propelling a space vehicle from planet to planet around the entire Solar System indefinitely without using any additional onboard rocket propulsion. The trajectory profiles could be represented by sequences of the form $P_0 - P_1 - P_2 \ldots - P_n$ $(n \geq 2)$, where P_0 denotes the launch planet and P_n denotes the last planet (or destination) in the encounter sequence. Minovitch viewed each intermediate planet in the sequence as a moving gravitational field that interacts with the vehicle's mass to provide the necessary propulsive thrust relative to the Sun—as though it were originating from a powerful onboard rocket engine—to reach the next planet P_{i+1} in the sequence. Once the vehicle is launched onto its first leg (which could be a low-energy trajectory), all subsequent interplanetary trajectory changes are accomplished free-of-charge, without any onboard rocket propulsion, by a series of controlled gravitational interactions with each passing planet. Control is accomplished by selecting various planetary approach trajectories obtained by solving the corresponding Restricted Three-Body Problem. As an added benefit, these propulsive sources are located at the planets, where the vehicle could conduct close-up reconnaissance observations and carry out various planetary measurements.

This concept represented a completely new propulsion theory for interplanetary space travel, fundamentally different from the classical theory based on the reaction engine. In this propulsion theory, the energy required to change a trajectory comes from the unlimited orbital energy of a passing planet, not from the limited chemical energy stored in propellant tanks. Since this energy can never be exhausted, the vehicle can be propelled around the Solar System with major trajectory changes indefinitely. Instead of viewing planetary perturbations as annoying disturbances of two-body motion, Minovitch viewed them as the primary propulsive forces in three-body motion.

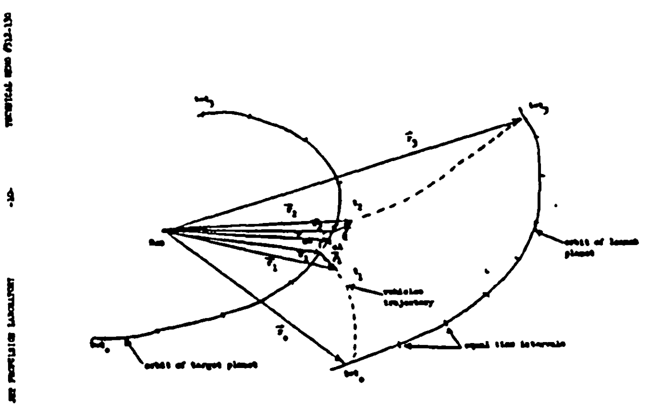

Figure 5 Reproduction of page 10 of Minovitch's August 23, 1961, JPL paper illustrating the radical trajectory changes Minovitch contemplated for free-fall trajectories, which led to his discovery of gravity-propelled interplanetary space travel.

At this juncture, Minovitch knew that he was onto something important, and he decided, on his own initiative, to write up this research as an official JPL Technical Memorandum, dated August 23.[73] Figure 5 is a reproduction of page 10 from this paper, illustrating the radical trajectory changes Minovitch contemplated for free-fall trajectories by utilizing the gravitational influence of a passing planet.

Minovitch described his concept of gravity-propelled interplanetary space travel and the analytical details necessary for circumventing the much more difficult system of Three-Body Problems associated with the propulsion concept on pages 38-44 of this JPL paper. He illustrated the concept by giving a *seven-planet* gravity-propelled encounter sequence involving the inner and outer planets of the form Earth-Venus-Mars-Earth-Saturn-Pluto-Jupiter-Earth (page 39, Reference 73). This obscure paper is significant for two reasons. First, it presented the first practical numerical solution of the Restricted Three-Body Problem (as represented by the most practical application, free-fall interplanetary trajectories through the Solar System); secondly, and even more important in the history of space travel, it introduced a radically new propulsion concept, that made the entire Solar System accessible to exploration by relatively small chemical rockets. It shifted the basic technological means for achieving interplanetary space travel from huge launch vehicles and exotic reaction engines to a relatively inexpensive planetary approach, using guidance systems and conventional chemical rocket propulsion.

In explaining this unusual, unauthorized independent research, Minovitch recalls:

> This was nothing new to me. I conducted similar unauthorized independent research on two prior occasions when working as a temporary summer employee in 1959 at Research Chemicals Corporation (located in Burbank, California). This involved the theoretical determination of the magnetic susceptibility of rare-earth compounds[74] and their crystal structure from x-ray diffraction data.[75] At that time, my supervisor viewed these unexpected scientific papers as important contributions to the ongoing research at the laboratory and I received much praise and recognition for writing them. However, my motivation was not to receive praise or recognition, but to conduct and report original research for the pure love of doing it. My second paper was considered by Dr. Sten Samson (a research consultant from CalTech working with Dr. Pauling) to be sufficiently important to publish in a professional journal, but I never found the time to publish this one. But I do think it probably resulted in my being invited to work in Dr. Linus Pauling's laboratory at CalTech the following summer in 1960.

From these prior experiences, Minovitch believed that conducting unauthorized independent research at JPL during the summer of 1961 (much of it on his own time) would be received with the same favorable attitude, but it was not.

In retrospect, it seems unlikely that anyone at JPL took the paper very seriously (It was distributed throughout JPL's entire Systems Analysis Section). In the first place, at that time Minovitch was only a temporary summer employee, who had worked on trajectories for only two months. Secondly, the possibility of achieving space travel throughout the entire Solar System without onboard rocket propulsion (other than that required for the initial launch) was such a radical idea in 1961, and it was contrary to the established principles of space travel (based on the reaction engine), that some JPL engineers believed it violated the law of conservation of energy. As pointed out above, Euler's approximate solution of the Restricted Three-Body Problem required the conservation of orbital energy, regardless of any planetary encounter. Minovitch's patched conic approximate solution had a less complicated mathematical formulation than Euler's and, thus, looked quite simplistic. Since Euler was one of the most famous mathematicians in history, it is reasonable to conclude that Euler's approximate solution would be regarded as more accurate than Minovitch's. On the other hand, at that time project-oriented trajectory researchers were not generally familiar with Euler's work and the analytical details of the Three-Body Problem. Gravitational perturbations appeared in mission studies as relatively minor (but important) disturbances of one-way interplanetary trajectories that were handled by integration/iteration differential correction methods.[59]

Clarke was not entirely receptive to Minovitch's independent research. But in addition to the formal Technical Report he wanted Minovitch to write, Clarke also initiated a "Request for Programming" to Jim Scott's programming group (in JPL's computing section), to have Minovitch's solution to the one-way problem programmed in FORTRAN, for use as a subroutine in Clarke's conic trajectory program.[67,69] Scott assigned a new employee in his group, named Raoul Roth, to do the actual programming, and Clarke wanted Minovitch to consult with Roth as needed, to ensure that this potentially complex job was finished before Minovitch left JPL to return to UCLA at the end of the summer. Clarke understandably believed that Minovitch's independent work interfered with his project-related task assignments, which it did.

Although Clarke was not receptive to Minovitch's independent research, or to the concept of gravity propulsion, and did not ask him to conduct further work along these lines, he did see some merit in the basic vector methods Minovitch developed to study general problems in astrodynamics. He asked Minovitch to apply his methods to determine the miss distances for conic trajectories, due to velocity errors that arise at the end of a rocket propulsive maneuver. Minovitch studied this problem and presented a solution in a third paper dated August 28.[76]

After he completed this third paper, there was insufficient time remaining, in that Summer of 1961, for Minovitch to prepare the first paper as an external JPL Technical Report as Clarke directed. However, another paper, written by James Jordan on the determination of one-way interplanetary conic trajectories, using Lambert's equations—with an analytic formulation essentially identical to Minovitch's, using some of Minovitch's results involving asymptotic limits (see pages 5, 6 in Reference 69)—was published a few years later.[77] It described the basic analytical methods adopted by JPL in the determination of one-way conic trajectories.

To investigate the practical possibilities of the gravity propulsion concept, a large computer program would have to be constructed in FORTRAN for numerical processing. Up to this time (August-September, 1961), Minovitch had no experience in FORTRAN programming. His attempts to persuade Clarke to initiate another Request for Programming to do this task (as had been done for the first memorandum's algorithms) were unsuccessful, even though Minovitch outlined in detail the logical construction of such a program and specified all the various mathematical equations. Minovitch did manage, however, to persuade Clarke to let him study round-trip free-fall trajectories on a small IBM 1620 digital computer at JPL during the two-week 1961 Christmas vacation break from UCLA.

It was during this time (December 1961) that Clarke told Minovitch that his analysis and ideas were fundamentally incorrect, because he believed (along with many other trajectory researchers) that the orbital energy of a free-fall space vehicle moving in interplanetary space without rocket propulsion has to remain constant because of the law of conservation of energy, regardless of any planetary encounter. As Minovitch recalls:

> At that time, Clarke had in his office a stack of computer outputs from runs of Earth-Venus and Venus-Earth interplanetary one-way conic trajectories corresponding to various launch and arrival dates. Clarke used this output to illustrate the fact that the basic method for finding a round-trip free-fall trajectory to Venus was to find an energy match at Venus with the same arrival and departure date (a procedure that he evidently intended to automate by a computer search routine of previously generated data from his one-way trajectory calculations, page 10, Reference 78). Clarke pointed out that the vehicle's arrival energy at Venus (vis-viva energy C_3, which is equal to the square of the vehicle's asymptotic approach velocity relative to Venus) corresponding to the Earth-Venus trajectory had to be equal to the departure energy of the Venus-Earth trajectory. Clarke indicated that the energy had to be conserved since no rocket propulsion is used during the encounter. This, of course, was true and was part of my formulation. But Clarke assumed that this condition automatically implied that the vehicle's orbital energy must also be conserved relative to the Sun. I explained that this is not true and drew various velocity vector diagrams on his blackboard in an attempt to prove this point. I pointed out that the concept of conservation of energy applies to the total orbital energy of all the bodies in the

> Solar System, not to each individual body. But Clarke didn't appear convinced.

When Minovitch left JPL in December to return to UCLA, he was convinced that Clarke had no intention of programming his concept of gravity-propelled space travel for numerical investigation at JPL, because he believed that the concept violated basic laws of physics and, thus, was theoretically impossible.[73] Minovitch recalls his thoughts back then:

> Had Clarke believed that the orbital energy of a free-fall vehicle could be changed by a close planetary encounter, and had he indicated that my concept of gravity-propelled space travel was theoretically possible, and that more work on it would be conducted at JPL, then I suppose my involvement with it would have probably ended in December 1961. All of the computing facilities and programming expertise needed to conduct the investigation was at JPL. Aside from the theoretical aspects which I documented in my paper, I had, at that time, little expertise or time to conduct the investigation myself. But Clarke's rejection was deeper than merely a personal belief that the concept was probably impractical. In his mind, it was an impossible violation of a basic law of physics. Since my preliminary slide-rule calculations indicated that theoretically the concept offered an inexpensive means for giving mobility to an interplanetary space vehicle far greater than any onboard rocket propulsion system could provide while simultaneously enabling it to conduct multiple planetary reconnaissance, I decided it was too important to let the concept be ignored because of a basic misunderstanding of the principle of conservation of energy.

Having failed to interest Clarke (or anyone else at JPL) in the theoretical possibility of gravity-propelled space travel, Minovitch decided to take an accelerated course in FORTRAN programming at UCLA and to carry out the required numerical investigation himself.

At that time (January 1962), UCLA was the only university in the western United States that had a large IBM 7090 digital computer (one of the rare "supercomputers" of that era). Minovitch discussed his gravity propulsion ideas and proposed research project with Professor Peter Henrici, an applied mathematician in UCLA's Department of Mathematics, familiar with the Restricted Three-Body Problem, and with Frederick Hollander, Chief of 7090 computer operations at UCLA. With their recommendation, Minovitch was given unlimited access to the UCLA computer to conduct his research as the "Principal Investigator." This highly unusual large-scale research project—unusual because it was conducted by a non-faculty graduate student in mathematics—lasted from January 1962 to September 1964. It involved hundreds of hours of computing time on the UCLA computer.

In April 1962, Minovitch informed Clarke of his UCLA research project and asked him if he could arrange a test to be made on his UCLA FORTRAN program, that he constructed to investigate his gravity propelled space travel concept, using JPL's high-precision interplanetary trajectory integration/iteration differential correction program. Clarke indicated that such a test could be made and assigned the details to Gene Bollman.[79] Several tests were run on gravity-propelled trajectories of the form Earth-Venus-Mars-Earth, and they were extremely successful. Very rapid convergence to the exact numerical solutions were obtained, with several planets exerting gravitational forces on the vehicle continuously and simultaneously. Without Minovitch's initial approximations for the successive encounter trajectories (AB Figure 4), convergence would have been impossible. The tests demonstrated that Minovitch's analytic methods actually represented a general numerical solution of the N-Body Problem, as it applies to interplanetary motion through the Solar System. A review of the literature indicates that this was probably the first general numerical solution of the N-Body Problem (see p. 55, Reference 61).

By June 1962, it was clear that Minovitch's concept of gravity propulsion not only was possible, but that it had the potential of revolutionizing interplanetary space travel previously founded upon the reaction engine. Although JPL was initially unaware of the fact that Minovitch continued the development of his ideas at UCLA, and indeed never financially supported the project there, JPL did help Minovitch enlarge his research project in June 1962, by giving him access to its two 7090 computers. This quite unusual relationship between JPL, Minovitch and UCLA lasted from June 1962 through September 1964.

This period of pioneering trajectory research established the basic feasibility of the concept and eventually led to all of NASA's gravity-propelled missions: Mariner 10 Earth-Venus-Mercury mission (1973 launch), the Pioneer 10 and 11 missions (1972 and 1973 launches, respectively), the Voyager 1 and 2 missions (1977 launches), the low-launch-energy Galileo mission to Jupiter (1989 launch), the Ulysses mission to the Sun (1990 launch), and many more gravity-propelled missions and mission possibilities previously considered impossible with chemical rocket propulsion. The details of Minovitch's research project at UCLA and JPL during January 1962 to September 1964 will be discussed in subsequent papers.

Additional Historical Comments

Many papers, articles and books have been published[80-86] that attempt to explain gravity-propelled multi-planet space travel as an old concept originating from the work of Hohmann during the 1920s, or from the work of Crocco during the 1950s.[50,87] As we have attempted to document thoroughly and explicitly in this paper, such explanations are without foundation. In fact, we have demonstrated that the opposite is true: the concept of gravity-propelled interplanetary space travel was such a radically new idea in 1961, that the theoretical possibility was believed by many to represent a violation of a basic law of physics, and thus, a physical impossibility.

Gravity propulsion represents a fundamentally new propulsion innovation that did not have any incremental, slow historical evolution. Although it is true that both Hohmann and Crocco proposed sending a single interplanetary space vehicle past more than one planet, they both viewed planetary perturbations as annoying disturbances of their "ideal" constant elliptical paths. This can be demonstrated by examining their own publications.

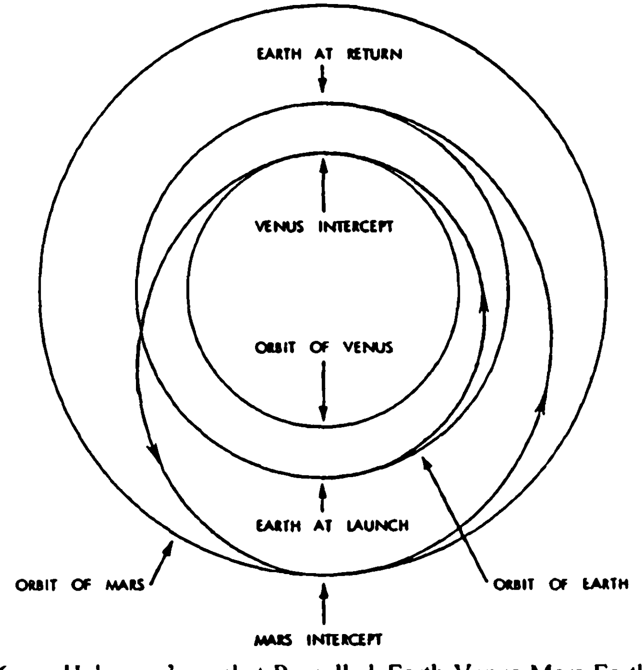

Figure 6 Hohmann's rocket-Propelled Earth-Venus-Mars-Earth multi-planet trajectory (1925).

Figure 6 describes Hohmann's Earth-Venus-Mars-Earth multi-planet trajectory (see pages 88-89, Reference 50). Basically, it consisted of three 180°

transfer trajectories around the Sun that required a total trip time of 1.5 years. When the vehicle reaches the first planet (Venus), rocket propulsion is used to change the trajectory so that it intercepts the second planet (Mars). At Mars, rocket propulsion is used a second time to change the trajectory so that it intercepts Earth. Hohmann recognized that the gravitational perturbations caused by each passing planet would influence the vehicle's trajectory. He viewed this influence as a serious problem and solved it by using additional rocket propulsion to cancel out the effect (see pages 88-89, Reference 50). This trajectory became known in the literature as "Hohmann's multi-planet Trajectory."[9] Unfortunately, due to the multiplying effects of the various propulsive maneuvers on the vehicle's mass ratio, it could be achieved only by using high-specific-impulse nuclear rocket propulsion.

Since Hohmann used onboard rocket propulsion to achieve this multi-planet trajectory, and since he viewed the planetary perturbations of Venus and Mars as annoying disturbances of the trajectory that must be corrected, it is clear that this trajectory cannot be regarded as a gravity-propelled multi-planet trajectory. Rather, it is actually a "rocket-propelled" multi-planet trajectory.

In 1956, Crocco discovered an unusual constant elliptical path that would, if it were not for the planetary perturbations, take a free-fall space vehicle past the orbits of Mars and Venus just as these planets arrived, and return it to Earth within a period of exactly one year.[87] This Earth-Mars-Venus-Earth multi-planet trajectory is illustrated in Figure 7. Like Hohmann, Crocco recognized that the gravitational perturbations of Venus and Mars would influence the trajectory.

In particular, they would destroy the constant elliptical resonant characteristic of the trajectory required to achieve the desired planetary intercepts. To solve this perturbation problem, Crocco used the Venus perturbation to cancel out the effect of the Mars perturbation and therefore obtain a final trajectory very close to his "ideal" precalculated, unperturbed, constant elliptical path. This trajectory became known as "Crocco's multi-planet Trajectory" (see pages 454-455 of Reference 13 for a further description of this multi-planet trajectory).

Unfortunately, any constant elliptical path around the Sun, with aphelion outside the orbit of Mars and perihelion inside the orbit of Venus, has a fairly high eccentricity and requires considerable launch energy. The fact that Crocco expended considerable analysis to obtain a trajectory close to his constant, perturbation-free ideal elliptical path (that required very high launch energy) demonstrates that he also viewed planetary perturbations as annoying disturbances of heliocentric free-fall conic motion.

Based upon what is found in the literature, we must conclude that these multi-planet trajectories of Hohmann and Crocco cannot be regarded as early examples of gravity-propelled interplanetary trajectories. Quite the contrary, these trajectories were designed to resist the basic ingredient that is the core of gravity-propelled trajectory design—planetary gravitational perturbations.

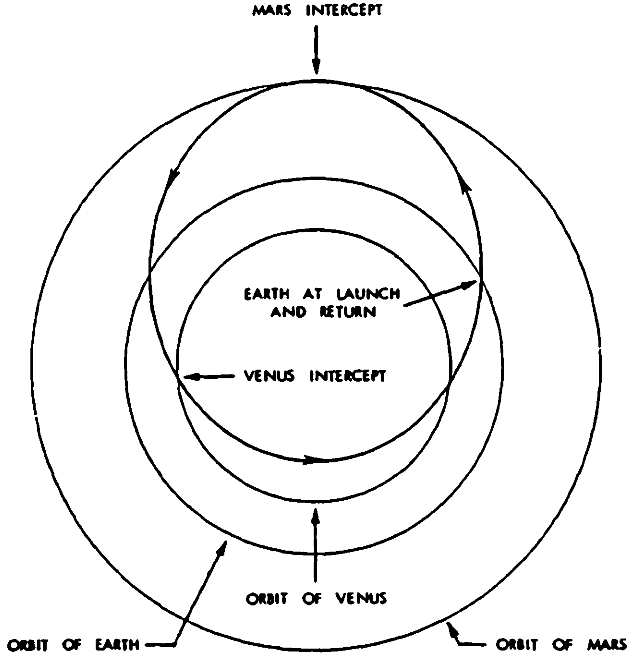

Figure 7 Crocco's constant elliptical path Earth-Mars-Venus-Earth multi-planet trajectory (1956).

Although Crocco's maneuver at Mars is accomplished gravitationally, and, strictly speaking, is therefore a "gravity-propelled maneuver," the overall trajectory cannot be regarded as a "gravity-propelled trajectory" for the simple reason that it was explicitly designed to achieve a trajectory identical to what would result if there were *no* planetary perturbations. Thus, introducing planetary perturbations in Crocco's trajectory design had absolutely no effect on re-reducing the overall rocket propulsion requirements (i.e., the very high launch energy remained unchanged).

Crocco's trajectory design was notable because the introduction of planetary perturbations in highly desirable round-trip multi-planet missions did not cause an increase in the propulsion requirements. The usual method of handling planetary perturbations in multi-planet trajectories was to cancel out their effect by onboard rocket propulsion, which significantly increased the propulsion requirements. Minovitch showed how multi-planet perturbations can be actually used to reduce the rocket propulsion requirements. This reduction—which came as a result of multi-planet trajectories—revolutionized interplanetary space travel.

A gravity-propelled multi-planet trajectory is a trajectory designed to use planetary gravitational perturbations as a basic vehicle thrust source to reduce the total onboard rocket propulsion requirements for a particular mission. In this propulsion theory, perturbations are not "utilized" to cancel out the effects of other perturbations to preserve, as in Crocco's trajectory, an eccentric constant elliptical path requiring very high launch energy. The required launch hyperbolic excess velocity V_∞ of Crocco's trajectory is 11.7 km/sec. (see page 235, Reference 87). If the mission calls for a non-stop free-fall round-trip trajectory that would pass both Mars and Venus, a gravity-propelled trajectory profile would have the form Earth-Venus-Mars-Earth. It would be designed so that the departing Earth-Venus leg is close to a Hohmann transfer, so that the required launch energy is near minimum. The Venus encounter would accelerate the vehicle to Mars (resulting in a major trajectory change relative to the Sun but provided free-of-charge by the orbital energy of Venus), and the Mars encounter would decelerate the vehicle back to Earth. The required launch hyperbolic excess velocity would be decreased from Crocco's 11.7 km/sec to about 3.5 km/sec. This represents a reduction in launch energy by a factor of 11.

Venus is encountered before Mars, because Venus has sufficient mass to increase the orbital energy of a near Hohmann low-energy Earth-Venus initial transfer trajectory, so that it could intercept Mars. Mars does not have sufficient mass to decrease the orbital energy of a near Hohmann Earth-Mars initial transfer trajectory, so that it could intercept Venus. These considerations are fundamental in designing gravity-propelled trajectories and, in this case, lead to a reduction in launch energy by over one full order of magnitude. Prior to 1961, no such analysis was presented.

In 1959, Harry Ruppe studied multi-planet trajectories with considerable numerical analysis. He essentially repeated the basic design methodology of Hohmann and Crocco (which he did not question), and he concluded that multi-planet trajectories required a great deal of rocket propulsion (see pages 189-190, Reference 9).

In 1962, a lengthy comprehensive analysis of interplanetary trajectories was conducted at Lockheed by a task force of ten well-known astrodynamicists and trajectory specialists, headed by Stanley Ross. It also included an investigation of multi-planet trajectories. (See Sec. 5, "Nonstop Trips Passing Both Mars and Venus: The Interplanetary Grand Tours," pp. 5-1 to 5-9, Reference 51). Based on theoretical and numerical analysis of these trajectories, it was concluded that multi-planet free-fall interplanetary trajectories required so much propulsive energy, that they were interesting only as "academic pastimes," which may be useful only after the development of nuclear propulsion (page 5-1, Reference 51).

Krafft Ehricke, another well-known astrodynamicist, also studied multi-planet free-fall trajectories in 1962 (see Sec. 9-9, "Interplanetary Flight Involving Several Planets," pp. 1058-1070, Reference 7.) He referred to these trajectories as "monelliptic" multi-planet trajectories, to emphasize that they are designed to be constant elliptical paths similar to Crocco's ideal constant elliptical path. However, unlike Crocco, Ehricke solved the planetary "perturbation problem" at each planetary encounter that tended to destroy the desired constant elliptical path, by applying brute force rocket propulsion to cancel out their effect. On page 1062 of this study, he states: "Perturbations by planetary encounters are assumed to be corrected, preferably while nearest to the planet, so that a heliocentric ellipse closely resembling the original ellipse is resumed by the time the vehicle is sufficiently removed from the planet" (Recall this is how Hohmann eliminated planetary perturbations). Ehricke's "tri-elliptic" and "bi-elliptic" multi-planet trajectories were connected segments of different elliptical paths, changed by applying rocket propulsion during the planetary encounters (i.e., they were rocket-propelled multi-planet trajectories).[7]

Clearly, the assertion that Hohmann's or Crocco's multi-planet trajectories are examples of gravity-propelled multi-planet trajectories is without foundation. The confusion may be the result of attempting to identify any previously proposed multi-planet trajectory as an example of gravity-propelled trajectories.

Not all researchers regarded gravitational perturbations as annoying disturbances of constant elliptical paths. The Russian space pioneers Y. V. Kondratyuk[88] and T. F. Tsander[89] both pointed out that the energy required to accelerate or decelerate a space vehicle beginning or ending an interplanetary journey can be reduced by taking advantage of possible satellite perturbations. But Tsander, who was aware of the fact that the orbital energy of a free-fall space vehicle can be changed relative to the Sun by planetary gravitational perturbations (see pages 278-289, Reference 89), never proposed utilizing this effect for reducing the basic propulsion requirements for traveling to another planet. Rather, Tsander regarded Hohmann's minimum energy trajectory for traveling to another planet as a fundamental "law" of space travel (see page 246, Reference 89).

In another example, D. F. Lawden in 1954, observed that a space vehicle sent to Mars or Venus can save energy by taking advantage of lunar or asteroid perturbations.[90] However, Lawden never suggested in this paper, or in any subsequent paper, the possibility of detouring the interplanetary path of a space vehicle to pass around a nearby intermediate planet to propel the vehicle gravitationally to a more distant target planet, thereby reducing the launch energy ordinarily required to reach the distant planet via the usual direct-transfer Hohmann trajectory. In several publications, Lawden states that it is not known how gravitational perturbations can be used to reduce the rocket propulsion requirements for space travel (see page 181, Reference 91; page 170, Reference 92; and page 52, Reference 15), and he never published any paper addressing the question.

When Lawden stated that "it was not known how gravitational perturbations can be used to reduce the rocket propulsion requirements for space travel," he probably had the unsolved Restricted Three-Body Problem in mind, because finding a numerical solution to this problem is a prerequisite for analyzing how gravitational perturbations can be used to reduce the rocket propulsion requirements of space travel; prior to 1961 no such solution existed. In his later publications, Lawden described the famous "Hohmann" trajectories as the "optimal" minimum energy interplanetary transfer trajectories, and he omitted any discussion regarding the possible use of gravitational perturbations.[16,17]

These remarks are not intended to detract from the outstanding contributions of these researchers, but, rather, to show that the concept of gravity-propelled interplanetary space travel did not originate with them (The popular terminology "gravity-assisted" or "swing-by" trajectories did not appear in the lexicon of space travel until 1964, as will be described in a subsequent paper). Ehricke himself believed that the concept of gravity-propelled space travel (which he called swingby trajectories) originated in the 1963-64 period.[93]

The Significance of the Gravity Propulsion Concept

To understand the significance of the breakthrough in interplanetary space travel represented by gravity propulsion, it is important to understand NASA's original plans for exploring the Solar System. These plans were based upon the underlying technical foundation of astronautics—propulsion technology, in particular—that existed at that time. Prior to 1961, it was believed that, for all practical purposes, the only method for propelling a space vehicle from one planet to another was by means of expelling mass via Newton's Third Law of Motion (a reaction engine).

The amount of mass (i.e., propellant) that a vehicle must expel to achieve a given velocity change is given by K. E. Tsiolkovsky's well-known "rocket equation":

$$M_i/M_f = \exp(\Delta V/u) \qquad (2)$$

where M_i and M_f denote the vehicle's mass before and after (respectively) expelling an amount of mass $m_p = M_i - M_f$ as propellant, with exhaust velocity u to generate the velocity change ΔV. It follows from this equation that if the desired ΔV is high, the exhaust velocity u must be very high to avoid unreasonably large mass ratios M_i/M_f.

Unfortunately, because of fundamental laws of thermodynamics, it is not possible to construct a chemical rocket engine having an exhaust velocity greater than approximately 4.70 km/sec. Consequently, it was taken for granted that any interplanetary mission requiring high velocity changes (such as manned Mars missions, unmanned deep space missions to the outer planets, solar probes or out-of-ecliptic missions) would automatically require very advanced nuclear and/or electric propulsion to enable the vehicle to deliver a reasonable payload.[20-29]

Since their inception in the late 1950s and early 1960s, nuclear propulsion and electric propulsion were destined for a long developmental effort.[94] The original U.S. planetary exploration program involved the exploration of only two planets, Venus and Mars, under project Mariner, using the chemical Atlas/Centaur launch vehicle.[34-39] The original (not outer-planet) Voyager missions called for the exploration of Venus and Mars, using the much larger Saturn/Centaur launch vehicle.[34-38] The higher energy missions had to wait until the required advanced propulsion systems were developed.

The invention of gravity propulsion changed the entire situation. Compare, for example, Reference 35 and page 5-1, Reference 51, written before the invention, to References 95 and 96, which were written by the same authors after the invention. Indeed, since research on electric propulsion over the past 30 years has not produced a viable system (primarily because of extremely low thrust-to-weight ratios), and since such a system is not expected to become available for several more decades, it can be accurately stated that the exploration of most of the Solar System has been enabled by the gravity propulsion concept.[94]

The following table is a list of essentially all of the basic interplanetary missions using classical Hohmann direct-transfer trajectories and their corresponding launch energies (V_∞^2), trip times, and the total number of these missions actually carried out over the past 30 years. The gravity-propelled trajectory profiles corresponding to these classical Hohmann missions are given to illustrate, quantitatively, how the invention of gravity propulsion changed the foundation and technical capabilities for space travel, which was previously based upon the reaction engine.

Table 1

INTERPLANETARY SPACE TRAVEL

Classical Direct-Transfer Hohmann Trajectories versus Gravity-Propelled Trajectories

Mission (Classical Trajectories)	Launch Energy $V_\infty^2 (km/sec)^2$	Trip Time (Years)	Number of Missions	Gravity Propelled Trajectory Profiles	Launch Energy $V_\infty^2 (km/sec)^2$	Total ΔV(km/sec) (Gravity Propulsion)	Trip Time (Years)	Energy Reduction Factor	Name of Mission
Earth-Mercury	56.6	0.289	0	Earth-Venus-Mercury	12-16	4-5	0.4-0.6	.21-.28	Mariner 10
Earth-Venus	6.3	0.400	7	-	-	-	-	-	-
Earth-Mars	8.7	0.709	6	Earth-Venus-Mars	12	4-5	0.8-1.4	None	-
Earth-Jupiter	77.2	2.732	0	$E-P_1-P_2-\ldots-P_n-J$	12-16	8-14	3-5	.16-.21	Galileo
Earth-Saturn	105.9	6.053	0	Earth-Jupiter-Saturn	77	12-16	≈3	≈.7	Pioneer 11; Voyager 1
Earth-Uranus	127.2	16.041	0	Earth-Jupiter-Uranus	77	12-16	≈5	≈.6	-
Earth-Neptune	135.7	30.646	0	Earth-Jupiter-Neptune	77	12-16	≈6	≈.5	-
Earth-Pluto	139.5	45.600	0	Earth-Jupiter-Pluto	77	12-16	≈6-7	≈.5	-
-	-	-	-	E-J-S-U-N(combination)	95	20.7	12.01	-	Voyager 2
-	-	-	-	$E-P_1-P_2-\ldots-P_n$(general)	12-16	Unlimited	Unlimited	-	-
Earth-Mars-Earth	16-25	2.7	0	Earth-Venus-Mars-Earth	12-17	6-7	1.2-1.8	.5-1.0	-
Earth-Venus-Earth	12-16	≈1	0	Earth-Venus-Mars-Earth	12-17	6-7	1.2-1.8	None	-
Crocco(E-M-V-E)	137	≈1	0	Earth-Venus-Mars-Earth	12-17	6-7	1.2-1.8	0.09-0.11	-
Solar Escape:									
V_∞(sun) = 0 km/sec	155	-	0	Earth-Jupiter-Escape	77	12-14	-	≈.5	-
V_∞(sun) = 20 km/sec	290	-	0	Earth-Jupiter-Escape	77	12-18	-	≈.3	Pioneer 10
Solar Probe:									
Perihelion =.005 AU	800	0.180	0	Earth-Jupiter-Sun	77	8-14	2.5-3.0	≈.09	Ulysses
Impact	886	0.178	0	Earth-Jupiter-Sun	77	8-14	2.5-3.0	≈.08	-
Out-of-Ecliptic									
i=90°,e=0, a=1.0 AU	1,780	1 yr Per.	0	-	-	-	-	-	-
i=90°,e=.74,a=2.9 AU	2,350	5 yr Per.	0	Earth-Jupiter-90°	77	16-18	5 yr Per.	≈.03	Inter. Solar Polar
				$E-P_1-P_2-P_3-$ (never ending cycling trajectories for manned interplanetary transportation systems)	12-16	Unlimited	Unlimited		

In this table, the column labeled Energy Reduction Factor is equal to the ratio of launch energies required for gravity-propelled trajectories and the corresponding Hohmann trajectories given by V_∞^2 (gravity propulsion)/ V_∞^2 (Hohmann). In view of the exponential nature of the rocket Equation (2), any significant reduction in launch energy translates into a very significant reduction in mass ratio (and hence propellant). For example, in the case of missions to Mercury, a gravity-propelled Earth-Venus-Mercury trajectory reduces the required Hohmann "minimum launch energy" from 56.6 km^2/sec^2 to about 14 km^2/sec^2. A reduction of this magnitude allows the payload mass to be almost triple that of Hohmann payloads. Or, for the same payload mass, a Hohmann Earth-Mercury launch vehicle would have to be approximately three times more massive than a corresponding Earth-Venus-Mercury launch vehicle.

The effect of launch energy reductions becomes extremely important when very large payloads are required, as in the case of manned interplanetary missions. For example, as described above, a multi-planet free-fall round-trip mission, passing both Mars and Venus, using a classical constant-elliptical-path Crocco trajectory, requires a launch energy of 137 km^2/sec^2. This can be reduced to about 13 km^2/sec^2, using a gravity propelled Earth-Venus-Mars-Earth trajectory. Assuming an all-chemical propulsion system, this enables the mission to be carried out with a single NOVA-type launch vehicle, instead of several NOVA launch vehicles (or a battleship-size Sea Dragon as shown in Figure 3). The cost of the mission would be significantly reduced.

The table also demonstrates quantitatively how much gravity-propelled trajectories can reduce trip times to the outer Solar System. For example, a gravity-propelled trajectory of the form Earth-Jupiter-Neptune will reduce the Hohmann trip times by a factor of 5 while, at the same time, reducing the launch energy by a factor of 2. By replacing the initial direct-transfer Earth-Jupiter leg by a gravity-propelled trajectory of the form Earth - P_1 -P_2 . . . - P_n - Jupiter, where P_i = Venus, Earth or Mars, the required launch energy to Jupiter can be reduced by a factor of 5. This type of gravity-propelled trajectory is being used in the Galileo mission to Jupiter. Since Jupiter is the key to the Solar System, these trajectories are very important. In fact, by using these trajectories, any region in the Solar System can be reached by an initial low energy Earth-Venus transfer requiring a launch energy of about 12 km^2/sec^2.

The column labeled ΔV(gravity propulsion) refers to the total velocity change generated by gravity propulsion relative to the Sun. For example, in the case of Voyager 2, the total velocity change was 20.7 km/sec.[97] If this change were accomplished by an onboard chemical rocket engine (with storable propellant such as hydrazine) instead of gravity propulsion, the required propellant mass would be on the order of 4 million metric tons (The corresponding mass ratio is approximately 5 x 10^6). It would be physically impossible to carry out such a mission using chemical rocket propulsion.

There is one unique and beautiful operating characteristic of gravity propulsion that sets it apart from all other vehicle propulsion concepts. Relative to an inertial frame of reference, the dynamic propulsive force F that determines the motion of a vehicle with mass m is given by Newton's well known equation

$$F = ma \tag{3}$$

where a denotes the vehicle's acceleration dv/dt relative to the reference frame. In gravity propulsion, the propulsive force F is simply the local gravitational force of attraction exerted by a passing planet given by equation (1). Since the vehicle's gravitational mass m_2 is equal to its inertial mass m (Newton's equivalence principle), the vehicle's mass term cancels out when equations (1) and (3) are combined to give the vehicle's equation of motion

$$\frac{dv}{dt} = G\frac{m_1}{r^2} \tag{4}$$

The solution of this equation is a hyperbola relative to the planet, but it is not a hyperbola relative to the Sun. Thus, unlike any other vehicle propulsion system, the vehicle's true acceleration path, relative to a Sun-centered reference frame, is independent of its mass.

When the vehicle is being propelled relative to the Sun (moving inside the planet's sphere of influence), its orbital energy relative to the Sun can be changing by essentially unlimited amounts depending upon its mass. The greater the vehicle mass, the greater the energy exchange—and the energy exchange is accomplished directly between the vehicle and the planet. The planet's orbital energy therefore represents an essentially infinite energy reservoir that propels the vehicle. Since the vehicle mass is canceled out in the equation of motion, it does not matter how massive the vehicle is once it is launched onto the first leg. The greater the vehicle mass, the greater the propulsive force. The propulsive force increases automatically with vehicle mass, as given in equation (1). Hence, in this gravity propulsion concept, there is no practical upper limit to the amount of "free" energy available to carry out interplanetary trajectory changes, given that the planets are so massive. Gravity propulsion therefore represents the most powerful known propulsion concept that can be used to propel anything moving in space. Minovitch therefore constructed a theory of interplanetary space travel that derives its basic propulsive energy from the essentially inexhaustible orbital energies of the various planets.

Since this propulsion theory is independent of vehicle mass, the ultimate application will involve propelling vehicles having very high mass. Such vehicles could be huge free-fall, gravity-propelled space liners, shuttling unlimited numbers of passengers around the Solar System, from planet to planet, in a vast interplanetary transportation network using planetary orbital energy for propulsion that could operate forever. The trajectories of these vehicles are indicated in the table by the unending planetary encounter sequence E - P_1 - P_2 - . . .

Since the innovation of gravity-propelled interplanetary space travel had such an impact on the exploration of the Solar System, one might ask why the concept wasn't recognized prior to 1961. As involved as many researchers were with their studies of planetary perturbations, why didn't they see the potential?

Without question, the answer lies in the fact that the concept involved a very difficult, theoretically-unsolvable mathematical problem, namely the Restricted Three-Body Problem of celestial mechanics. And the only alternative—a general numerical solution—did not exist prior to 1961.

But the concept of gravity propulsion involved not only finding a solution to this problem, but also to a much more difficult problem: it involved determining a trajectory having a series of successive planetary encounters. Each encounter trajectory around each planet in the series had to be precisely determined, so that its gravitational influence would catapult the vehicle on to the next planet, etc., often with radical changes in its path. The gravity propulsion concept, therefore, required a practical numerical solution for a system of Three-Body Problems.

Since the Three-Body Problem in itself was so difficult, trajectory analysts (who are usually not mathematicians) were probably repelled by problems dealing with gravitational perturbations. Those who were tasked to invoke state-of-the-art theory to uncover new interplanetary trajectory possibilities were most likely not looking to gravitational perturbations to come to their aid.

The literature also suggests a possible wide-spread assumption among trajectory engineers that the orbital energy of a free-fall vehicle relative to the Sun must remain constant, regardless of any planetary encounter. Such an assumption would obviously inhibit the discovery of gravity propulsion on more fundamental grounds. These, we feel, are probably the underlying reasons why the gravity propulsion concept was not recognized prior to 1961.

Summary

After Minovitch developed his vector techniques as a powerful mathematical tool for dealing with trajectory problems in three-dimensional space (during the summer of 1961), he was attracted to the Restricted Three-Body Problem as a mathematical challenge. He invented (or discovered) the concept of gravity propulsion as a result of this research. He solved the mathematical problem by studying the physical situation and developing a method for decoupling the system of Three-Body Problems into a larger system of Two-Body problems. And, with the aid of his vector methods, he obtained a solution trajectory as a series of "patched-conic" approximations, that was sufficiently accurate to allow the approximation to converge to an exact numerical solution in a numerical integration/iteration differential correction process, using the exact differential equations of motion corresponding to the real situation, where various bodies in the Solar System exert gravitational forces on the vehicle simultaneously. This mathematical breakthrough was the crucial development behind the innovation because, without a practical mathematical solution to investigate the concept quantitatively, the concept was useless.

Minovitch refers to his innovation as "gravity propulsion" or "gravity-propelled" trajectories, rather than "gravity-assisted" or "swing-by" trajectories, to emphasize that this concept enables a vehicle's trajectory to be radically changed relative to the Sun, as though it were propelled by a powerful high-thrust rocket engine with infinite specific impulse. The fact that a planet is always in the vicinity when this propulsive thrust is applied, is a beautiful but incidental side benefit that allows the vehicle to conduct planetary reconnaissance as it passes. Notice that, since the application of conventional rocket propulsion is always most effective when a vehicle is passing closest to a planet, the application of gravity propulsion occurs when conventional rocket propulsion would occur. Thus, in this respect, gravity propulsion is similar to conventional rocket propulsion. But unlike conventional rocket propulsion, gravity propulsion does not require any propellant, and it is independent of vehicle mass.

An important element in the development of gravity propulsion was the high speed digital computer. When Goddard developed his liquid rocket reaction engine for propelling a vehicle, his research laboratory consisted of chemical elements and means for generating propulsive thrust by igniting and maintaining a sustained chemical explosion. Minovitch's laboratory, in contrast, was a facility equipped with a high speed digital computer, which he used to devise a method for converting planetary orbital energy into propulsive thrust for a passing space vehicle. Although these thrust-generating mechanisms are completely different, they provide the same effect—vehicle propulsion.

In 1961, when the mainstream of space propulsion planning was approaching the economic, if not physical limits of scale and power, Michael Minovitch introduced a new philosophy of space propulsion. Instead of attempting to conquer the Solar System with ever larger and more powerful rocket engines, he developed methods for harnessing its own orbital energy. In this philosophy, the

Solar System itself would provide essentially all of the propulsive energy required to explore it. Based more on an evolution of mathematics and digital computers than on engineering, it was an idea whose time had come.

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