Galaxy
SCIENCE FICTION

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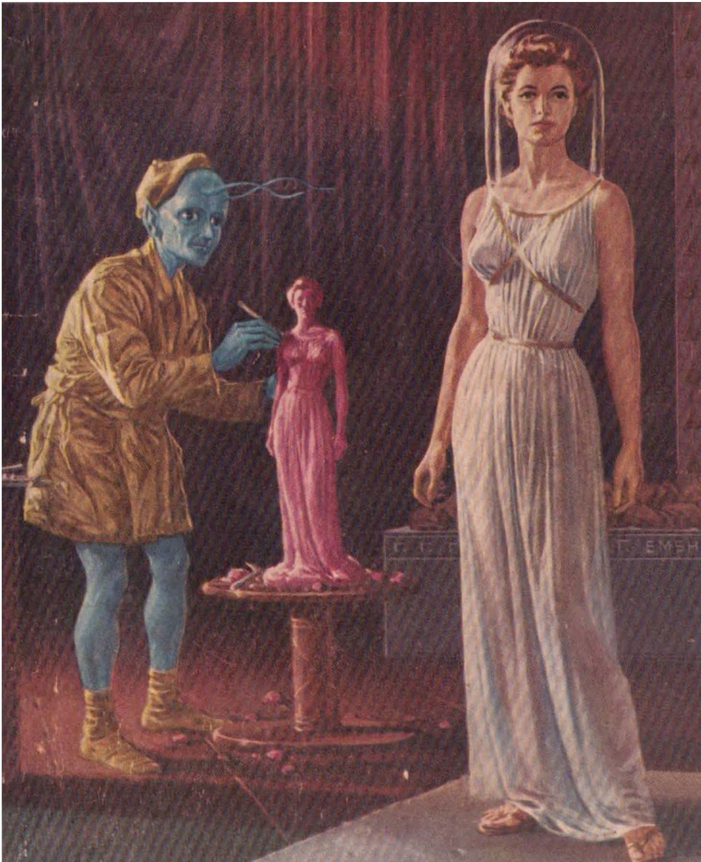

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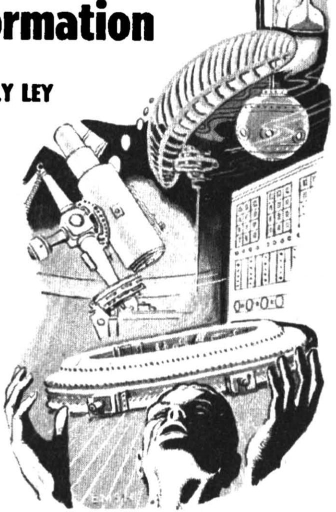
For Your Information

By WILLY LEY



THE 7-CORNERED POLYGON

I HAVE yet to compare notes with other columnists, but from my own experience it seems that long letters can often be answered completely with a sentence or two, while short queries might require a book. Of course, every science editor is plagued by letters of the type which ask, "Please explain the Theory of Relativity"—only eight years ago an institute received the perfectly serious request from a lady to "please send me what has been published about aviation"—but that is not the kind I have in mind. I am thinking of a short letter that came in some time ago and which consisted of precisely three sentences.

In the first sentence, the correspondent explained that his hobby was the making of scale models and that a classical chariot was the current project. In the second sentence, he said that the picture he owned showed seven-spoked wheels and could I tell him how to construct one. The third sentence thanked me for whatever help I could give.

This was easy. I made a sketch and said that this would do for his purpose, even though the construction was incorrect mathematically. Back came the reply that it had worked beautifully, but why did I say that this construction was incorrect? He had tried it several times and had come to the conclusion that the "in" in front of "correct" had slipped in by mistake. But if it hadn't, why was it incorrect?

Well, the answer is this column, for the simple question requires a treatise on the division of the circle.

LET'S begin with fundamentals. Using a protractor for dividing a circle isn't permitted—in geometry, that is. You can use a protractor if you want to cut up a pie or to make a wheel with a silly number of spokes; in short, for practical purposes. But in geometry you can use only two instruments, a pair of compasses and a straight edge. Furthermore, the straight edge must be used only for connecting points, *not* for measuring distances.

This strict rule has a good reason, even though beginners often have trouble understanding it. Ideally, you do all this in your head, drawing lines in the air with your finger; the lines on paper are merely a means of remembering (and communicating) what you have found by thinking. The straight lines just show which point is supposed to be connected with which other point. The circles "measure" since every point along the periphery of a circle has the same distance from its center.

A protractor and other mechanical devices are "forbidden" because they will furnish information which did not exist in your head *first*. You are not supposed to read off an angle of 30°; you are supposed to find it by reasoning.

For example: the sum of the three angles of a triangle is 180°; hence, each angle in an equilateral triangle must be 60°, and 30°

is half of such an angle. Or: a right angle is 90°; if I construct an equilateral triangle in a right angle, the difference between the right angle and the triangle must be 30°.

Now let's go on with the problem. Fig. 1 shows the ordinary hexagon, constructed, in this case, by first halving the circle by means of a straight line going through its center, and then using the compasses with the same opening that was used to draw the original circle from both ends of the diameter, points 2 and 5 in the diagram. Or you can do it without drawing a diameter first, by simply starting at any one point of the periphery with the compasses and going around. By halving the angles, you obtain the points for the 12-cornered polygon. In the diagram, the usual method of halving was not used, since in this construction you can halve one of the 60° angles—the one formed by points 1 and 6 with the center of the circle—by erecting a vertical line on the diameter.

By jumping every second point of the hexagon, you obtain an equilateral triangle, and by doing this twice in succession, you obtain the figure shown in Fig. 2, the Star of David (which produces a hexagon in its center). Another way of arriving at the 12-cornered polygon is shown in Fig. 3. Instead of starting with a hexagon and halving every angle, you begin with two diameters of the circle forming right angles at the center. Then you use the compasses with the same opening used for the original circle from these four points (Nos. 3, 6, 9 and 12 in the diagram) in the manner shown in the right half of the diagram.

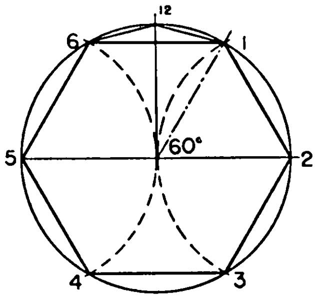


Fig. 1

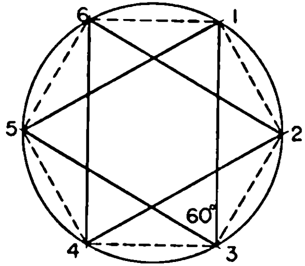


Fig. 2

The result is a figure sometimes called the Lilac Blossom (indicated at points A and B) and also the 12-cornered polygon.

THERE is still another way of constructing an equilateral triangle in a circle. This consists of drawing a radius of the circle, halving the radius and erecting a vertical line on the halfway point. The distance from the halfway point to the periphery of the circle is one-half of the side of the equilateral triangle, indicated by the points A, 2 and B in Fig. 4.

But it so happens that this half side is *very nearly* the side of a 7-cornered polygon. The difference is quite small, amounting to 17/10,000th of the radius of the circle; if you have a circle with a radius of 40 inches, the difference is just about 3/32nd of an inch. Such an approximation is good enough for seven-spoked wheels, model or full scale, but it is only an approximation. To construct the true side of the 7-cornered polygon with compasses and straight edge is impossible. The same holds true for the 9-cornered, 11-cornered and 13-cornered polygon, to mention only a few cases.

Don't waste your time trying. You probably will find approximations galore, but no true construction.

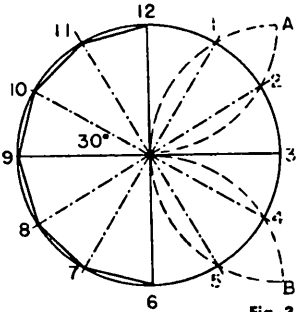


Fig. 3

In fact, there are only a few series which can be properly constructed. So far I have dealt with the one which I think of as the "hexagon series," which produces, beyond the hexagon, polygons with 12, 24, 48, 96, etc., corners. Another may be called the "square series," which is based on the square derived from two diameters at right angles to each

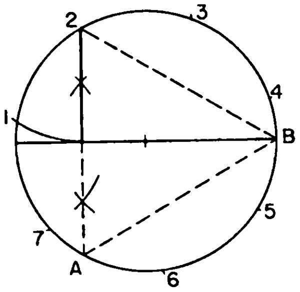


Fig. 4

other (Fig. 5) and leads to polygons with 8, 16, 32, 64, etc., corners.

Another one is the "pentagon series" (Fig. 6 and 7) which, interestingly enough, does not really begin with the pentagon but with a 10-cornered polygon The method is shown in Fig. 6. When you draw a smaller circle inside the first circle, the diameter of the small circle is equal to the

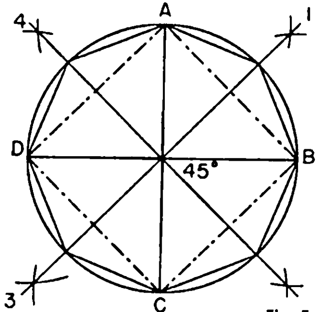


Fig. 5

radius of the large circle. Then you connect the center of the small circle to the point marked "3" in Fig. 6. The distance from point 3 to the periphery of the small circle is the side of the 10-cornered polygon. By jumping over every second point of the 10-cornered polygon, you obtain the pentagon, and by jumping every second point in the pentagon, you obtain the "magic" five-pointed star with a smaller pentagon in its center (Fig. 7). The "pentagon series," of course, leads to polygons with 20, 40, 80, 160, etc., corners.

Since a full circle has 360°, the angle required for a 15-cornered polygon is 24° and that can be constructed in an interesting manner. The angle of the equilateral triangle is 60°. The angle of the 10-cornered polygon is 36°. And 60 minus 36 = 24. The actual

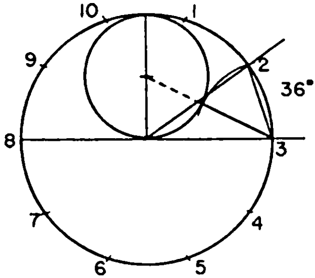


Fig. 6

construction is shown in Fig. 8. This naturally leads to polygons with 30, 60, 120, etc., corners by simple halving of the angles.

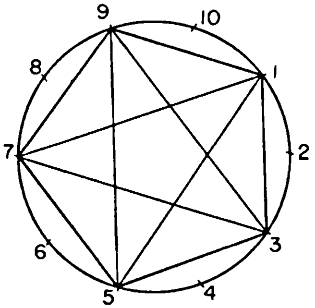
For more than twenty centuries, these remained the only polygons that could be constructed, even though people through all these centuries kept looking for more. They were especially interested in the 7-cornered polygon because seven is supposedly a magic and holy number. They also devoted much effort to the 9-cornered polygon, but for no special reason than that it just seemed to be simple.

Nobody, to my knowledge, ever wondered whether the possible and impossible constructions might both be covered by some law which one may discover.

IT was in 1796—we even know the date: March 30th—when a 19-year-old student discovered that law. His baptismal name was Johann Friederich Carl Gauss, but later he signed his work Carl Friedrich Gauss.

One of the consequences of the discovery was that it was possible to construct a 17-cornered polygon. It is not as simple a job as the ones discussed. In fact, the explanation would take up as much room as I have for my whole column, so that I can only say here where it may be found: in F. Klein's *Famous Problems of Elementary Geometry* (Hafner, New York, 1950). The 17-cornered polygon naturally leads to polygons with 34, 68, 136, etc., sides, and by using a method similar to the one for the 15-cornered polygon, you can also construct a 51-cornered polygon (from triangle and 17) and an 85-cornered polygon (from pentagon and 17).

Well, what is Gauss's law? What is possible?



Gauss's first reasoning was that only polygons with an *odd* number of sides need to be considered. The even-numbered polygons are just the result of halving angles; the whole "square sequence" works that way, because you first halve a full circle, then the semi-circle and so forth. Gauss then found and *proved* that the odd-numbered polygons

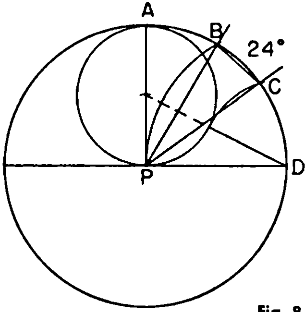


Fig. 8

which can be constructed are the same as the Fermat primes. Therefore you can construct:

$2^1 + 1 = 3$ (triangle)
$2^2 + 1 = 5$ (pentagon)
$2^4 + 1 = 17$
$2^8 + 1 = 257$
$2^{16} + 1 = 65,537$

All of them actually have been constructed, the last one of this series only once. And you can also construct odd-numbered polygons where the number is the product of the multiplication of two Fermat primes, hence the 15-cornered polygon (3 x 5) and the 51 and 85-cornered polygons (3 x 17 and 5 x 17, respectively). Likewise the 3 x 257-cornered polygon, etc., etc., should be possible.

But 7, 9 and 13 are not.

ANY QUESTIONS?

How does a living cell know when to stop growing? Does the action of gravity have anything to do with the decision of an ameba to split?

David L. Osborn
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The answer to this question is simple, if probably unsatisfactory: we do not know. There is obviously a complex of factors involved that will have to be unraveled slowly and patiently. Nor is it certain that an answer which would apply to a cell that is part of a unit (say, in the leaf of a tree or in the muscle of an animal) would also apply to cells that are "free," as, for example, the red corpuscles that circulate in our blood.

In the case of the ameba, cited by you, one might think of gravity as the determining factor if the ameba were a dry-land creature. But since it lives in water, which supports every portion of it, all the responsibility can't be placed on gravity alone.

I remarked in one of my books that one of the biological research projects after the completion of the space station might well be to have unicellular plants and animals, like bacteria and amebas, grow in a zero-g condition with plenty of food around and see what happens. I can't predict what will happen, but I do believe that whatever is going to happen will furnish the main clue for an answer to your question.

The Milky Way stretches from the northeast to the southwest. Does this give any indication as to our position in our galaxy? And another question: is the plane of the Moon's orbit around the Earth constant or does the

Moon travel in various paths around us?

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Comanche, Oklahoma

As to the first question: no. But from other observations, it has been concluded that our sun and its family of planets are between two-thirds and three-quarters of the total possible distance from the center. In other words, it is at least twice as far from the Sun to the center of the Galaxy as it is from the Sun to the rim.

The orbit of the Moon happens to be unusually complicated and its calculation ranks among the more difficult problems. The minimum distance is 221,463 miles, the maximum distance 252,710 miles. The inclination of the plane of the Moon's orbit, compared to the ecliptic, the plane of the Earth's orbit, also varies. It can be as little as 4 degrees and 57 minutes of arc and as much as 5 degrees 8¾ minutes of arc. This means that there is a kind of doughnut-shaped volume of space around the Earth which contains all the possible positions of the Moon.

I would like to know if the planets closer to the Sun rotate faster or slower; is there any relationship between the distance from the Sun and the speed of rotation of planetary bodies?

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I wish I could be perfectly sure that you actually mean "rotation" (around the axis) and not "revolution" (around the Sun). As regards the latter, there is a very precise and clear-cut relationship between the orbital velocity of a planet and its distance from the Sun. The closer they are to the Sun, the faster they move — Kepler's Third Law — the farther away from the Sun, the slower.

For example, Venus proceeds at the rate of 21.7 miles per second, the Earth moves at 18.5 miles per second, Mars 15 miles per second, Jupiter 8.1 miles per second, while Saturn proceeds at the comparatively leisurely pace of "only" 6 miles per second.

But there is no relationship between the period of *rotation* of a planet and its distance from the Sun.

Mercury, the innermost planet, completes one rotation in the same time it needs for one revolution, namely 88 days. The rotational period of Venus is not known, but it seems to be about two of our weeks. Earth and Mars have periods of 24 hours and 24 hours 37 minutes,

respectively, while Jupiter, Saturn and Uranus rotate quite fast. The figures are, in the same order, 9 hours 55 minutes, 10 hours 14 minutes, and 10 hours 40 minutes. Neptune needs 15 hours and 40 minutes, while the rotation of Pluto is as yet unknown.

These figures look as if there might be a relationship between the size of a planet and its diurnal period because Jupiter, with an equatorial diameter of 86,700 miles, is the biggest planet and also has the shortest period. Saturn, Uranus and Neptune are slower in the order mentioned, which is also the order of decreasing diameters, namely 71,500, 32,000 and 31,000 miles, respectively. Likewise, when you consider the inner planets, Earth has a shorter diurnal period than either Venus, Mars or Mercury, and Earth is the biggest of the inner planets.

This sounds like an intriguing idea until you look at the *masses* of the planets rather than their *diameters.* Jupiter does rotate faster than Saturn, the difference being 20 minutes. But Jupiter's mass is equal to 317 Earth masses, while Saturn's is equal to 95 Earth masses. Saturn rotates faster than Uranus (25 minutes difference), but the difference in mass between these two is even more impressive, for Uranus has only 14.7 Earth masses. Neptune needs 5 hours more than Uranus, but while the handbook will tell you that Uranus has a somewhat larger diameter than Neptune, the same handbook will tell you that Neptune is the more massive of the two, having 17.2 Earth masses.

That these differences in rotational period are not in proportion to the differences in mass is especially clear in the case of the inner planets. In round figures, Earth is ten times as massive as Mars, yet the Martian period is only 37 minutes longer.

In short, there is no relationship between period of rotation and distance from the Sun, but in general it may be said that, at least in our solar system, the more massive planets tend to rotate faster than the lighter ones.

—WILLY LEY