

History of Rocketry and Astronautics

**Proceedings of the Seventh and Eighth History Symposia
of The International Academy of Astronautics**

Baku, U.S.S.R., 1973

Amsterdam, The Netherlands, 1974

Kristan R. Lattu, Volume Editor

R. Cargill Hall, Series Editor

AAS History Series, Volume 8

A Supplement to Advances in the Astronautical Sciences

IAA History Symposia, Volume 3

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AAS Publications Office
P.O. Box 28130
San Diego, California 92128

Affiliated with the American Association for the Advancement of Science
Member of the International Astronautical Federation

First Printing 1989

ISSN 0730-3564

ISBN 0-87703-307-2 (Hard Cover)
ISBN 0-87703-308-0 (Soft Cover)

*Published for the American Astronautical Society
by Univelt, Inc., P.O. Box 28130, San Diego, California 92128*

Printed and Bound in the U.S.A.

Chapter 3

DEVELOPMENT OF THE PRINCIPAL PROBLEM OF INERTIAL NAVIGATION*

L. I. Tkachev[†]

Spacecraft navigation may be provided by an Inertial Navigation System (INS) in the absence of external information. This is exemplified by the U.S. Apollo Project Lunar Module, which is equipped with the two sets of INS of different types: a) the Strapped Down System, b) the INS with accelerometers, stabilized in the direction of the primary navigation star (Ref. 21).

Speaking of inertial navigation systems, we would like to mention the name of Professor Charles Stark Draper, the highly respected President of the International Astronautical Academy. A quarter century ago, when many scientists, including outstanding scientists, considered blind navigation to be fantasy (Ref. 22), Dr. Draper had implicit faith in the possibility of inertial navigation. He went through a long and hard road in the development of INS for lunar vehicles, and his efforts were crowned with triumph.

The development of modern, precise INS became possible with solutions to theoretical and engineering problems. The theoretical problem of inertial navigation was to eliminate methodical errors of INS occurring in the case of the direct application of accelerometers for navigational purposes.

The engineering problem, or "the problem of technology," was to reduce the sensors' instrumental errors.

The present paper treats only the principal (theoretical) problem in a historical aspect. This problem derives from the physical fact that in a spacecraft the accelerometer measuring an apparent acceleration can't differentiate an absolute and gravitational acceleration from centripetal and Coriolis ones. The equivalence of accelerations manifesting in weightlessness (zero-G) was taken into account [by Konstantin Tsiolkovsky] at the outset of astronautics (Ref. 1).

The principles of inertial navigation can be described by the laws of classical mechanics and even by one of its sections: rigid-body kinematics. However, it his-

* Presented at the 7th History Symposium of the International Academy of Astronautics, Baku, 1973.

† Academy of Sciences, USSR.

torically appeared that the possibilities of using the laws of mechanics for inertial navigation were not at first realized.

The determination of the position (location and orientation) of a movable frame, arbitrarily relocating and rotating in the space of any gravitational field in the absence of external information, may be assumed as the general task of inertial navigation (Ref. 24).

The historical criterion: If one correct algorithm for the general task had been developed the solution of the principal problem would have been completely established.

In accordance with this criterion, the ideas of inertial navigation may be considered to go through the next three historical stages:

1. The prehistory: The embryonic period of the development, from the desire for autonomous estimation of a vehicle's position by means of inertial devices (1903-1905), to the realizing of the principal problem, as a whole (1940-1942).
2. The qualitative leap: The determination of ideal equations of inertial navigation for the general task; that is, the solution of the principal problem of inertial navigation (1942-1944).
3. The period of contemporary inertial navigation: Its first period of a real history (1944-1956), the suggestion of many versions of inertial navigational systems, both analytic and semi-analytic ones; and the second half of the three-decade development of inertial navigation, from the beginning of the Sputnik era (since 1957). This latter phase can be completely analyzed later.

PREHISTORY OF INERTIAL NAVIGATION

Sometimes in the literature, the prehistory period of the development of inertial navigation is expanded in centuries, going back to Newton. However, Siegfried Reisch writes: "Newton had precisely as much to do with INS as Faraday with radar and Democritus with the A-bomb" (Ref. 23).

In our opinion, the prehistory period started from the statement of the problem of deducing a vehicle's position on the basis of the inertial information. This problem was first identified by Mervin Carry (USA) in 1903 (Ref. 2). He put forward the idea of gyroscopic quasi-astro-orientation; however, he didn't take into consideration the very important point of determination of the vertical on a moving vehicle.[†]

Raingard Vussov (China, 1905) suggested applying gyros, accelerometers and integrators to determine the velocity and the distance covered by a vehicle (Ref. 3).

* That's why the works of well-known scientists such as Academician A.J. Ishlinsky, Prof. Ch. Broxmeyer and others are not considered here.

† On the other hand, we don't consider here theories of indicating the vertical, unless the problem of deducing the vehicle's geographical coordinates was simultaneously included. So, we do not consider Schuler's paper "Phys Zeitschr" N 16. 1923, although it aroused a great response in the applied gyroscope theory. In accordance with S. Reisch's opinion (Ref. 23), we may say that the study of Schuler's work is not necessary for the successful development of INS.

Thus, the problem was formulated. Inertial sensors had been known, it was next required to develop the correct algorithm of using their initial information. But Vussov assumed as a basis, a homogeneous gravitational field, that's why his scheme was suitable only for the hypothetical Flat Earth, Fig. 1. Johann Boykov (Austria, 1911) suggested (Ref. 4) applying a gyropendulum instead of Vussov's free gyro. But he didn't take into consideration that such a simple gyro horizon had ballistic deviations $\beta(t)$ resulting in an error ΔL of the dead reckoning:

$$\Delta L = \int_0^t dt \int_0^t G\beta(t)dt \tag{1}$$

We will not analyze these schemes in detail. They are not free from methodological errors.

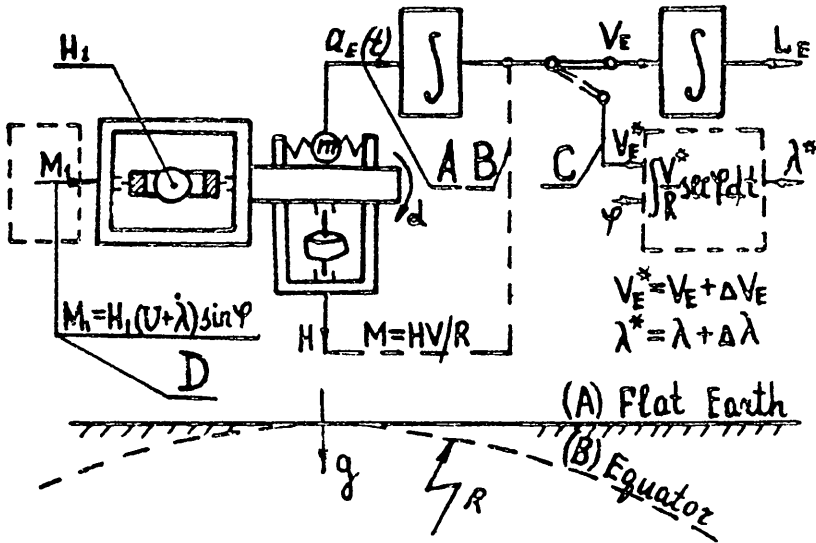


Figure 1-A/B/C/D

Only two particular tasks of inertial navigation were correctly solved between Vussov's time and World War II. Exceptional cases of a vehicle's movements are considered in these tasks: motion through space in a fixed vertical (in Newton field), and motion through space in a fixed "great circle" arc (in the central field).

The Polar Axis Task

The first task belonged to Esnault-Pelterie (France, 1930). He investigated the movement of a rocket along a non-rotating vertical beam (Ref. 5). Such conditions, for instance, are required for a meteorological rocket launched along the Earth's Polar axis from the Earth's south pole. Pelterie turned his attention to the influence of gravitation (G). He pointed out that the accelerations d^2z/dt^2 having been deter-

* One of the founders of rocket technology, H. Oberth, in his book "Die Raketten zu den Planetenräumen", München, 1923, spoke of double integration of accelerations, but he didn't take into account the influence of gravitation.

mined by the readings $\alpha(t)$ of a vertical accelerometer had a methodical error due to Max Born's relations:

$$d^2z/dt^2 = \alpha(t) - G_z \quad (2)$$

which expresses Einstein's Equivalence Principle.

E-Pelterie suggested corrections, taking into account Newton's dependence of gravitation upon the radius:

$$G_z = G_{RR^2}/z^2 \quad (3)$$

This idea was stated in the section devoted to the treatment of errors (Ref. 5, p. 189) and until then hadn't been noticed by readers. However, we can state that E-Pelterie pointed the way to the solution of the Polar Axis Task. It may be considered to be the first step in the inertial navigation theory.

The Equator's Task

The second elementary task was first investigated by E.B. Levental* and L.M. Coffmann (USSR, 1930-1936).

The idea of the "integrated correction" was suggested by them in patents (Ref. 6). Due to "the integrated correction," the idea of the scheme (Ref. 3), Fig. 1-A, becomes suitable for the hypothetical task of a vehicle's displacement along the line of the Earth's equator (Fig. 1-B).†

The inventors (Ref. 6) pointed out that, in the equator's task, their scheme was free from ballistic deviations, and hence, it had no methodological errors expressed by the formula (Ref. 1). They also pointed out that error in the inaccurate, initial leveling leads only to oscillations in the readings of the covered distance.

The Sphere's Task

Certainly, Levental and Coffmann (Ref. 6) didn't intend to limit the displacement of a vehicle only by the equator's line. They wanted to provide for navigation on the Earth's surface, taken as a sphere. But they neglected the convergence of meridians and it seemed to them that two identical pairs of integrators could provide the calculations of the two geographical coordinates, latitude and longitude.

In fact, however, this scheme wasn't suitable for the "sphere's task." The same "integrated correction" was assumed as a basis of the patent (Ref. 7) which had been granted to I. Boykov in 1928-1934. In addition, Boykov took into account Merkator's extension of the longitude's scale as the function of the latitude (Fig. 1-C). He foresaw for the azimuth orientation the possibility of applying not only a gyrocompass but a "velocity compass," i.e., a directional gyro equipped with the correction for the true and apparent motion of the Earth (Fig. 1-D).

* We should like to emphasize that in the literature on inertial navigation, the name of Levental was first mentioned in Reference 15, and the names of Vussov and Coffmann in Reference 24.

† The same scheme was submitted by S. Reisch in the 1945 (Ref. 12).

Boykov, however, neglected Coriolis accelerations, and thus, he also didn't develop the correct algorithm for the "sphere's task." Other versions of the dead-reckoning schemes were then suggested by L.M. Coffmann (1937, Fig. 2-B), by L.I. Tkachev, A.M. Letov and V.A. Vengerov (1939, Fig. 2-C), and by E.B. Levental (1940).

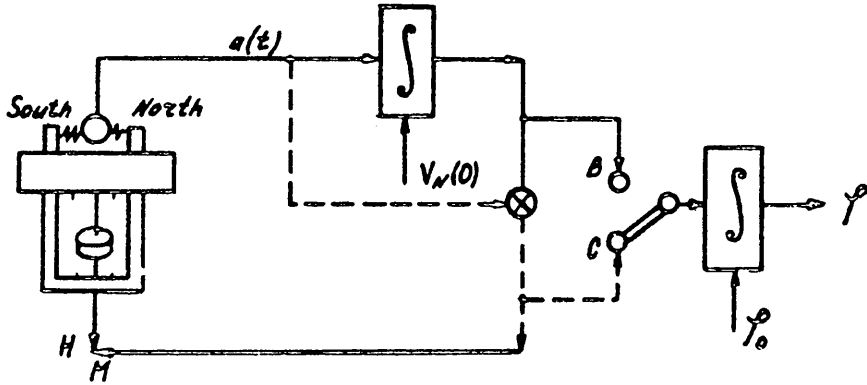


Figure 2-B/C

Furthermore, three other papers (Ref. 8, 9, 10) at that time investigated some of the above-mentioned theories. While studying (1934) Carrie's scheme (Ref. 2), S.I. Kudrevich turned his attention to instrumental errors (Ref. 8). He pointed to a great influence of gyro drift. A thorough understanding of the physical nature of the processes gave him the possibility of using this idea for the scheme (Ref. 6). The expression of the error of the dead-reckoning due to gyro drift ($\omega_{gp} = \text{const}$) for the "Equator's Task" is known now, but at that time it had not been written.†

$$\frac{\Delta L}{R} = \omega_{gp} t - \frac{\omega_{gp}}{\nu} \sin \nu t \quad \nu = \sqrt{G/R} = 1.24 \cdot 10^{-3} \text{ [cek}^{-1}] \quad (4)$$

Kudrevich pointed out that the component accumulating in time existed. In 1936 Kudrevich and N.I. Sygachev warned Coffmann against applying secondary integrators for the real development of a complex navigational system for ships--"Universal Orientator" (UO). He advised to provide the functions only of a gyrocompass, a gyro horizon and a directional gyro. Coffmann agreed with this idea, but soon he stopped work in this area.

In 1937, Kudrevich's successors again added secondary integrators to the scheme UO, then called UO-37. The new people who took charge of the work asked a scientific consultant, Prof. B.V. Bulgakov, to make a mathematical analysis

* The scheme Fig. 2-C was considered in 1958, M. Schnerb, La navigation, N. 3, 115 (1958).

† See, F.E., Stratton A., Proc. IEE, Part B, 105, 9, 226 (1958).

of a device. He submitted his paper, "The theory of a precise gyroscopic system 'UO-37,'" in 1938 (later published (Ref. 9) in 1969).

The techniques of the applied gyroscope theory, well-developed later in the book, were used here (Ref. 9a) The main techniques were as follows: the application of damping, turning off episodically; the study of ballistic deviation, except Coriolis accelerations; limitation of a vehicle's movements only by the sphere's surface with the central gravitational field; feasibility of external information (readings of a log).

The objectives of studying the applied gyroscope theory, were basically the development of mechanisms for the indication of navigational directions of the vertical and the meridian (Ref. 9).

Bulgakov pointed out that the methodical errors being inherent in the scheme "UO" are inevitable for any design perfection (Ref. 9, p. 26). He also pointed out that ballistic deviations in the indication of the vertical had numerical value, with applications for maritime use. As to methodical errors of dead-reckoning, Bulgakov didn't study them, but he pointed to a great instrumental error. It meant the confirmation of Kudrevich's warning against applying secondary integrators for a ship's system, taking into consideration the existing level of gyro drift.

In 1939, A.M. Letov, L.I. Tkachev and V.A. Vengerov studied the same scheme "UO-37" for application in aeronautics (Ref. 10). The methods of the applied gyroscope theory were still used at that time. The schemes suggested in 1939 were not yet free from methodological errors.

The transition from the original scheme (Ref. 6, 7) to the present semi-analytic INS, for example, "the free-azimuth scheme" Fig. 1-D (the "a" and "c" types according to Broxmeyer classification) can be done by means of applying additional computing devices. This development came later, and before the year 1942, none of the suggested schemes gave the correct solution to the task of navigation on a sphere.

Historically it appeared that the sphere's task was solved only after the solution of the general task--the task of determining location and orientation of a movable frame, arbitrarily relocating and rotating in the space of the real (non-central) gravitational field.

THE QUALITATIVE LEAP: THE SOLUTION OF THE GENERAL TASK AND THE ESTABLISHMENT OF THE FOUNDATIONS OF INERTIAL NAVIGATION THEORY

The solution of the general task of inertial navigation was first brought to the notice of scientists in L.I. Tkachev's report at the Leningrad University on January 18, 1943. The report was "On the Theory of the Spatial Orientation in a Blind

* Unfortunately, there was misrepresentation of Bulgakov's remark in the paper (Ref. 9).

Flight by Means of Accelerometers and Gyros."* The possibility of inertial navigation in any gravitational field without methodological errors was formulated and proved in this report. The analytical method of determining the vertical under arbitrary movements of a vehicle in the arbitrary gravitational field was also suggested. The report was approved by the scientists of the Leningrad University, the Moscow University, the Baumann Institute of Technology, the Moscow Power Engineering Institute and the Zhoukovsky Academy.

Some time later, "On the Technological Facilities of the Development of an 'Absolute Auto-Navigator'" was presented at the Moscow Power Engineering Institute. This work and the first Tkachev's report were submitted as a doctoral dissertation (Ref. 13) to the Baumann Institute of Technology on February 5, 1944.

Let's analyze this paper (Ref. 13) which summarized the results of the investigations of the 1940-1943 period. The foundations of the inertial navigation theory were presented in the paper (Ref. 13).†

The theorem of "Spatial Orientation" constitutes the main part of the whole paper. It was formulated and proved in the report. The theorem was as follows: "A device consisting of accelerometers and gyros being able to indicate an absolute angular velocity's vector of some movable frame and the summary inertial and gravitational vector of a mass concentrated at the frame's center makes it possible to deduce the characteristics of the frame's position and motion as the function of time in the absence of an external information in a given gravitational field under the given initial conditions, with the accuracy of an instrumental error alone". (Ref. 13, p. 91) This theorem became essential for the further development of inertial navigation. It may be called, "The Theorem of Existence" of inertial navigation.

Inertial navigation systems were put into consideration by means of the inertial navigation theorem in the canonical form. It indicates an absolute angular velocity vector " ω " of sensors' frame arbitrarily rotating in a space and the vector of an apparent acceleration " α " of the frame's vertex. These vectors are indicated in the form of the projections on the frame's axis. The canonical form gave great possibilities for determining different versions of INS and studying the properties of INS that are common from different types.

The analytical method was applied for the solution of the General Task of inertial navigation. The determination of the vertical's direction by means of calculations is made simultaneously with the analytical evaluation of a vehicle's position (Ref. 13, p. 94).

The equations of inertial navigation were then developed. They represent the system of differential equations for deducing the navigational state of a movable frame by means of complete inertial information (Ref. 13, p. 103).

* The documents relating to this report, on January 18, 1943, are kept at the Institute of the History of Natural Science and Engineering (Academy of Science, USSR).

† The solution of the Principal Problem is misdated in the literature by naming the year 1946. J. Slater said: "Even at the beginning of 1946 there was no clear understanding of the inertial navigation principles" (Ref. 20). H. Hellman reported: "It was but in 1946 the members of the German team in Fort-Bliss (USA), developing Reisch report (1945), formulated the perfect inertial system's principles for the rotating Earth" (Ref. 22).

The possibility of getting the versions of equations by means of utilizing different systems of the navigational coordinates and types of orientation parameters was shown (Ref. 13, p. 93). It proved (Ref. 13, p. 103) that the solution of the suggested system of the equations existed. Using the modern terminology (Ref. 8, 24), the idea of the equations of inertial navigation may be generally expressed:

$L_t = MTN(\tilde{I}_0^t / L_0)$, i.e., at the moment t navigational state L_t with the original value L_0 on the location M with the navigation chart N is the multidimensional functional T of the complete inertial information $\tilde{I}_0^t \equiv (\tilde{\omega}_0^t, \tilde{\alpha}_0^t)$. Notations $L \equiv (\vec{\chi}, \vec{v}, \vec{\psi})$, $\vec{\chi}$, \vec{v} , $\vec{\psi}$ are the vectors of location, velocity and orientation: $\tilde{\omega}_0^t$, $\tilde{\alpha}_0^t$ are the hodographs of the vectors $\vec{\omega}$, $\vec{\alpha}$.

The equations of inertial navigation (Ref. 13, pp. 103, 93) are the set of the equations of location (the gravitational field G), the equations of orientation, and the equations of equivalence ($d\vec{v}/dt = \vec{\alpha} + \vec{G}$).

The functional $T(\tilde{I}_0^t / L_0)$ is the mapping of the objectively existing principle of a complete inertial information (Ref. 24, p. 100).

It showed the physical nature of inertial navigation on the real location in contrast to the widely-shared belief (Ref. 6, 7) of the possibility of determining location only by double integration of the readings of horizontal accelerometers.

It was also shown that the equations of inertial navigation were "ideal," which meant that the possibility of a complete algorithmical autocompensation of methodological errors in the absence of an external information was set. Thus, the solution of the principal problem of inertial navigation was found (Ref. 13, p.94).

The theory of inertial navigation was strictly formulated, which demanded the correct application of the mathematical methods of rigid body kinematics. All the velocities, accelerations, rotations and vertical displacements of a movable system were used in the equations. It took into consideration that the gravitational field is non-central and the Earth isn't spherical (Ref. 13, p. 103). Dissipation in the system was not allowed.

Thus, the methods of the modern theory of inertial navigation were put into practice. They are different from the methods of the traditional applied gyroscope theory (Ref. 9a).

The three main methods of inertial navigation determined the following types of analytic INS:

1. The Strapped-Down System, (Ref. 13, p. 107).
2. The analytic system with the platform controlled by the independent non-precise indicators of the vertical and the meridian (Ref. 13, p. 99).
3. The analytic INS with the platform kept near the horizon and the meridian controlled by means of its own navigation computer (Ref. 13, pp. 137-140).

The semi-analytic INS with the platform stabilized by the meridian (Ref. 32, pp. 169, 171) making the calculations of Coriolis accelerations was given as the special case of the canonical INS (1945). The INS with the analytic determination

of the vertical with the platform of accelerometers stabilized in the stars' direction (Ref. 15) was also given as a special case of the canonical INS (1948). Moreover, the idea of the platform stabilized by means of "an arbitrary compass and the horizon" (Ref. 13, pp. 99, 107) anticipated the further investigations of the semi-analytic INS with the orientation by an ideal gyrocompass (Ref. 16), and with the orientation by a free-azimuth gyro (Ref. 19, pp. 338-390) (Ref. 18, pp. 4-11). The algorithms of the versions of the analytic and semi-analytic INS result from the equations (Ref. 13, p. 103) as the special cases (Ref. 25, p. 193).

The function of the semi-analytic INS proved to be analogous to gyro-pendulous indicators of the vertical in the solution of the theoretical task of the Spherical Earth. A gyro-pendulum is designated as the system with the "integrated correction" (Ref. 13, pp. 67-69).

Thus, the analysis of the equations (Ref. 13, p. 103) shows that they are general as follows:

1. The INS's algorithms of different types result from the equations.
2. They compose the general method of the navigational information genesis on the basis of using the set of equations of orientation, equivalence and gravitation.
3. They suggest the fruitful idea of applying the mathematical methods of rigid body kinematics to the theory of navigation -- the idea gave rise to the development of new versions of algorithms.
4. The equations for estimating the INS's instrumental errors can be obtained from these equations by means of variations of the initial conditions and parameters.

On the whole, the work (Ref. 13) determined the qualitative leap in the development of the ideas of orientation without external fixtures.

Thus, if Vussov couldn't overcome the concepts of the hypothetical Flat Earth, and the other investigators (1923-1942) didn't go beyond the concept of the Spherical Earth (Ref. 3), the theorem of "Spatial Orientation" eliminated all the principal limitations and gave rise to the development of the modern inertial navigation.

The work (Ref. 13) was thoroughly investigated by outstanding scientists such as B.V. Bulgakov, S.S. Tikhmenev, D.U. Panov, G.O. Fridlender.

All those interested in inertial navigation from the first steps of its development became familiar with this work in the USSR. The general equations of the canonical INS and the particular algorithms resulting from them (Ref. 29, pp. 117, 133-135, 140) are presented in the modern complete monographs on the inertial navigation theory.

The equations represented here--either in the form of evolution or multiplicity, in the form of matrices and vectors, applying various coordinate or notation systems--are described in accordance with (Ref. 13, pp. 103, 93). Unfortunately, in the literature there is infrequent mention that the general equations of inertial navigation became worthy of science as early as 1943.

Progress in the INS theory was made in 1948 when the property of oscillation in the readings of the Strapped-Down System in the non-central gravitational field

was discovered by L.I. Tkachev. This property, and the idea of the Strapped-Down System (SDS) itself, were revealed to a wide range of readers in a paper (Ref. 15) in 1949. The idea of the analytic determination of the vertical in INS with the platform of accelerometers stabilized in a star's direction was also suggested there. The operation algorithm of such INS is developed as a particular case of the SDS algorithm:

$$\omega_1 = \omega_2 = \omega_3 \equiv 0$$

During the development of the Apollo lunar landing project, two types out of the whole variety of well-known INS schemes were chosen for application. They were (Ref. 21) the SDS, and the system with the star-stabilized platform of accelerometers. The ideas of these very schemes were presented in Reference 15.

The real development and the application of these two types of INS in lunar vehicles drew theorists' and engineers' attention. Some versions of orientation algorithms were suggested for Strapped Down Systems. It should be noted that the application of the so-called Poisson's equations is not a new suggestion, as this form of writing kinematic equations is automatically obtained by matching a combination of expressions (1-1) Table 1, when there is no requirement to define orientation angles (Ref. 25, p. 68).

Many publications and scientific conferences, held during the three decades after the origin of the idea of the Strapped Down System, gave rise to its application on the Earth.

THE PERIOD OF CONTEMPORARY INERTIAL NAVIGATION

The chronosequence of the inventions of specific types of INS (free from methodological errors) during the fifteen years after the establishment of the canonical INS, was as follows:

1942-1944. L.I. Tkachev. a) The Strapped Down System for navigation in a space of any gravitational field using the inertial coordinate system; and the same for the non-central field with the geocentric coordinate system (Ref. 13); b) The analytic system with the platform controlled by the independent non-precise indicators of the vertical and the meridian for navigation in the non-central gravitational field using a geographical coordinate system (Ref. 13); c) The analytic INS with the platform kept near the horizon and the meridian controlled by its own navigation computer (Ref. 13).

1945. L.I. Tkachev proposed a particular case of the previous scheme: the semi-analytic INS stabilized by the meridian making the calculations of Coriolis accelerations for navigation in the non-central gravitational field (Ref. 32).

1947. Ch. Fox proposed the semi-analytic INS with the passive stabilizations of two gyrotachometers by an ideal gyrocompass on the "Spherical Earth" (Ref. 16).

1948. L.I. Tkachev proposed the INS with the platform of accelerometers stabilized in the direction of a star and with the analytic determination of the vertical (Ref. 15).

1948^{*}-1951. I.M. Lisovich, G.I. Vasilyev-Lulin, B.E. Chertok proposed the version of a geometric type of IS for aircraft movement along a loxodrome (Ref. 28).

* The dates of the work remain to be confirmed by documents.

1946*-1952. The Charles Stark Draper Laboratory (Wrigley, Houston, De'Lisle) proposed versions of a geometric type of INS for aircraft and ships (Ref. 17).

1952. L.I. Tkachev proposed a version of the geometric INS with the application of Boykov's rotor integrated accelerometer (Ref. 14B).

1953*-1956. "Autonetics" Co (J. Slater) proposed a free-azimuth system (Ref. 19).

1953-1956. L.V. Kondratyev submitted the general equations of the free-azimuth system, for the meridians' grid arbitrarily overturned on the "Spherical Earth" (Ref. 18).

The publications on the history of inertial navigation are brief in content and in some cases they deviate from the truth. The author will be pleased if the data contained in this paper proves of some use to historical science.

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