

LETTERS AND COMMENTS

Reply to Comment on ‘An analysis of Newton’s projectile diagram’

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Abstract. We reply to a criticism of our paper (1999 *Eur. J. Phys.* 20 59–66) made by Michael Nauenberg.

That a scientist could be aware of some consequences of his work, or even should be aware, is no guarantee that he is aware. Maxwell died of cancer at the age of 48 without realizing the full significance of his theory, particularly the production and detection of electromagnetic waves. He certainly could have realized this, since it was entailed in his equations. But he did not. Things are often clear only in retrospect.

Indeed, Nauenberg is acutely aware of this problem in reading Newton. In his meticulous and often brilliant reconstruction of Newton’s work [1], on which his present Comment [2] is based, Nauenberg continually uses phrases throughout such as, ‘Newton could have solved analytically’, or ‘Newton could have applied’, or ‘There is no direct evidence that Newton actually carried out such calculations’, or ‘Newton may well have discovered in this way’ [1, pp 221, 225, 229, 231, 232, 233, etc].

In his Comment [2] on our paper [3] Nauenberg certainly showed that Newton could have known that the South Pole is the maximum point a horizontal projectile can reach before it goes into orbit. As he shows, such information is contained in Book I, Proposition 45 of the *Principia*. We would agree that Newton should have noted this. The only question remaining is whether he did, in fact, realize this.

The fact is, the specific case of an inverse square force applied to the general central force law is buried in Newton’s text as a short

phrase. There is nothing there to indicate unequivocally that Newton fully understood the significance of his derivation. Certainly there is no reference to the projectile passage.

Also, still left unresolved is the problem of the passages in Definition 5 in both the *Principia* and in *On the System of the World* which imply, as we have shown, that Newton conceived of a spiral path striking the earth beyond the South Pole. Nauenberg’s argument in footnote 2, which is an attempt further to undermine our argument, contains a contradiction. As we pointed out in a footnote which was dropped from the original article because of space limitations on our footnotes, in the third edition of the *Principia* Newton deleted the phrase about the projectile ‘falling’, almost as if he had caught his error. But a closer reading, and a comparison of it with the phrase in the second edition, reveals otherwise. The passage in contention is where Newton (discussing projectiles being shot from the North Pole mountain) writes the following phrase:

till at last it should fall at the distance of 10, 30, or 90 degrees, or even might go quite round the whole earth before it falls; or lastly, so that it might never fall to the earth, but go forwards into the celestial spaces, and proceed in its motion in infinitum.

We have written the text in italic, except for the segments that Newton extracted from the third edition of the *Principia* which are written in roman. We assert that in the full second edition version the phrase ‘quite round the whole earth before it falls’ is the case of the projectile going around the earth and (say) terminating at the base of the mountain. Then, with further initial velocity, the projectile makes it back to the top of the mountain and hence into orbit, or, as Newton says, it goes ‘forward into the celestial spaces’. Hence in the context of the second edition ‘forward into the celestial spaces’ means into a closed orbit unimpeded by the Earth. In the third edition, although the ‘falling’ sub-phrase is eliminated, Newton still speaks of the projectile going ‘quite round the whole earth or lastly go forwards into the celestial spaces’. The case of a projectile going ‘quite round the whole earth’ is, we assert, a repetition of the spiralling of the projectile terminating at the base of the mountain—for the next sub-phrase begins, ‘or lastly’, indicating that this is the next, last, and a different case, indeed the one where it goes into orbit. So therefore the third edition changes only the language of the projectile passage (presumably for the relief of what Newton thinks is redundancy), not the meaning. A modern reader whose only acquaintance with this passage is from a reading of the third edition might be inclined to believe that Newton’s phrase ‘forward into the celestial spaces’ refers to the inevitable high-eccentricity hyperbolic paths that high-velocity projectiles take, but this would be an ahistorical and probably incorrect interpretation. We feel strongly that we cannot merely assume that Newton’s third edition is just a fixed-up version of the earlier editions, thereby freeing ourselves from the necessity of studying the earlier text. Sometimes you have to read the earlier editions to see what Newton was genuinely discussing later on.

Or put it this way: if it was really the case that Newton had recognized his error and wanted to eliminate the reference to the spiral, would he not have written in the third edition something like this: ‘10, 30, 90 and 180 degrees, or lastly go forward into’, that is, definitely and definitively stating his case? And, should he not have thus added, as we just did, the case of 180 degrees, if he knew that the South Pole was the maximum point for a projectile to strike the earth? Moreover, if Newton had found his error, would there not perhaps be other corroborating evidence (such as a more explicit statement of this in Proposition 45), rather than just the deleted phrase in Definition 5?

Accordingly, Nauenberg is contradictory. On the one hand, he argues that our interpretation of the passage in the third edition is wrong; that it shows Newton correcting his error. And, on the other hand, he says that Newton knew about the correct orbits in the first edition. But Nauenberg (and Newton) certainly cannot have it both ways.

Thus the puzzle remains: if Newton not only could and should but also did know of the 180 degree maximum projectile, why are there spiral passages in the texts?

We wish to thank Bruce Brackenridge and Michael Nauenberg for their helpful, open, and cordial discussions of these matters, even though we are not in full agreement with their conclusions.

References

- [1] Nauenberg M 1994 Newton’s early computational method for dynamics *Arch. Hist. Exact Sci.* 46 221–52
- [2] Nauenberg M 2000 Comment on ‘An analysis of Newton’s projectile diagram’ *Eur. J. Phys.* 21 L5–6 (preceding Comment)
- [3] Topper D and Vincent D 1999 An analysis of Newton’s projectile diagram *Eur. J. Phys.* 20 59–66